Math 329 Unit 3 Fall 2014

For each of the questions which involve figures, your unit3.m script should create a pdf of the figure. Use the command: `print(figure(1),'-dpdf','Q4b_figure.pdf')` The -dpdf can be changed to make jpeg(-djpeg), tiff(-dtiff) and many other formats. But for these assignments a pdf file is best.

Questions that involve work by hand can be turned in on paper or you can have the unit3.m file type the answers as part of the output. As before, please use the diary feature to create an output text file such as: `diary 'output_unit3.txt'` or something similar.

1. Use Lagrange's formula for polynomial interpolation with the three points $0, \frac{\pi}{2}, \pi$ for the function $f(x) = \sin(x)$ to estimate $\sin(1)$. (You can use paper for the formula and a calculator to estimate, or you can type Lagrange's formula into Matlab and compute the estimate that way.

   Now use the Matlab polynomial interpolation routines (polyval, vander and/or polyfit, etc) to estimate $\sin(1)$. Compare your answers – is this what you would expect? Why?

   Analytically find the minimum and maximum error bounds using the error estimate
   $E(x) = (x - x_1)(x - x_2)(x - x_3)f^{(n+1)}(\xi)/(n+1)!$.
   Compare the actual error to this error bound. Note: You DO know $f(x)$ in this case, but ξ is still only known to be within some range.

2. How can we check the accuracy of interpolation routines? One way to get a feel for it is to try them on known functions and see how they behave.
   Interpolate $x^3 * sin(x)$ on 100 evenly spaced points in $[0, 2\pi]$

   (a) linear interpolation: 100 points $\Rightarrow$ This will serve as our "true" solution and we can graph it using linear interpolation.

   (b) linear spline (linear interpolation): 5 (evenly spaced) points

   (c) 4^{th} order polynomial: 5 (evenly spaced) points

   (d) 24^{th} order polynomial: 25 (evenly spaced) points

   Interpolate each of these to the 100 points and create one graph depicting all three curves. Note for b) you can use interp1() with option 'linear'. For c) and d), you can use polyval() and polyfit().

   How do you interpret the error/warning message you get for the 24^{th} order polynomial? Briefly relate that message to ideas about polynomial interpolation discussed in class.

3. Compute the error of the interpolation at the 100 points we are using as our basis of comparison for the 4^{th} order and 24^{th} order polynomial interpolant above. (You don't need to print them out!) How can we describe this error? Calculate (and hand in) the norm of the error, using 1,2 and ∞ norms. Write a paragraph describing whether and in what ways each of these norms reflect the error you can see visually in the graphs.

4. Now we'll explore how the accuracy changes as the function gets less "smooth"?
   (a) Compute the following function values at 100 points in the given range.
   (b) Interpolate from 5 points using a 4^{th} order polynomial.
   (c) Interpolate from 10 points using a 9^{th} order polynomial.
   (d) Compute the norm of the residual error vector using the ∞ norm.
   (e) Plot the three interpolated curves with dots for the data points. Label the graphs.

$$x^2 \cos(x) \qquad [0, 2\pi]$$

$$abs(x) \qquad [-1, 1]$$

$$\log(abs(x^2 + \cos(x))) \qquad [0.1, 6.0]$$

   What features of the functions make interpolation more difficult? Write a few sentences describing the implications of these examples on using polynomial interpolation.

5. **Note:** This exercise uses the bisection.m routine you wrote from unit 1. Take a few moments to make any suggested changes to that routine. As you rediscover your own code, add comments as needed to remind you how to use it and how it works. (You will be using it again.) You should include bisection.m in the zip file submitted with this unit.

   Here we will turn the interpolation problem on its head. We wish to find the data point that will create a specific interpolated value. Find the polynomial interpolated value of a function at $x = 1.25$ if the function passes through the points (-4,-10), (0,-3), (1,1), (2,3), (5,6), (7,14). Suppose each data point indicates the effect of one "knob" on a machine. The x value denotes which knob and y the setting of that knob. The output of the machine is determined by the interpolated value $f(1.25)$. We would like to add one more "knob" to get the output to be 2. The "knob" corresponds to $x = 4$. Use polynomial interpolation and your bisection routine to find the correct "setting" (y-value with tolerance 10^{-10}) of the "knob" to get the output of the machine to be 2(with tolerance 10^{-10}).

   Note: You will have to think of a way to state the problem as a function you can give to the bisection routine. What is the input to the function (something you are looking for)? What is the output of the function (something you want to equal zero)? Once you figure out what the function is coding it shouldn't be too difficult.