Math 329 Unit 5 Fall 2014

Due: Wednesday, November 12

1. We have a model for the production of the new Eepoxee Mouthwash. Our production process involves the mixing of three compounds. These compounds tend to break down on their own, so it is important to create them in the same mixing process that produces the mouthwash. The production rate can be written as

$$p'(t) = k_1 xyz - k_2 yz + k_3 x^2,$$

where $p(t)$ is the total amount of mouthwash produced, k_1, k_2, k_3 are constants and x, y and z are (possibly negative) measures of the three compounds.

The three compounds are themselves created through a process we have modeled as:

$$x'(t) = a_1(1 - x) + a_2 y,$$

$$y'(t) = b_1 y(1 - x^2) - x - 0.01yp'(t),$$
$$z'(t) = c_1 z(1 - z) - zp'(t)$$

Our current values for the controlling parameters are: $a_1 = 0.423$, $a_2 = 1.012$, $b_1 = 0.553$, $c_1 = 0.500$, $k_1 = 2.512$, $k_2 = 0.734$, $k_3 = 1.222$. We would like to simulate the process to inform us as to why our production levels are so sporadic. This analysis should include graphs of all three compounds in addition to a graph of total production. The final analysis should also include programming code so that we can update parameter values and rerun the simulations.

2. Consider the scalar IVP $y'(t) = f(t, y) \quad y(0) = y_0$. Derive an $O(h^3)$ (on each timestep) scheme for simulating the solution using the multistep form

$$Y_{n+1} = y_n + ay_{n-1} + h\left[b_1 f(t_n, y_n) + b_2 f(t_{n-1}, y_{n-1})\right].$$

What is the order of the truncation error when simulating from $t = 0$ to $t = 25$?

3. Simulate two pendulums that are attached by a spring at the pivots so that when one swings the other tends to feel a force in that direction. The equations are:

$$\theta_1'' + f\theta_1' + \sin(\theta_1) = -k(\theta_1 - \theta_2)$$

$$\theta_2'' + f\theta_2' + \sin(\theta_2) = -k(\theta_2 - \theta_1)$$

Use initial conditions $\theta_1(0) = \theta_2(0) = 0$, $\theta_1'(0) = 1$, $\theta_2'(0) = 0$. Use friction constant $f = 0.1$ and coupling constant $k = 0.3$.

Plot the time profile of the variables θ_1(red) and θ_2(green) for time $0 \leq t \leq 30$.

4. Using the same simulation data, plot the phase plane for each pendulum on the same figure – red for the first and green for the second. Does this system have periodic motion?

5. Now set $f = 0$ and $k = 0$. Plot (and print) the phase plane of each pendulum. Why does only one show up? Does this system have periodic motion?

6. Increase k to 0.04. Simulate to $t = 400$ and look at the curves to get an idea of what the solution looks like (you don't need to save this figure. Now start the simulation again using the ending values (t=400) of one simulation to begin the next. Simulate for another 100 time units. Do not just type in the initial conditions for this second run. Use Matlab variables to get initial conditions from the previous simulation the restarted simulation. Those values are already in memory with better precision than you will type. Using the subplot feature, create a single figure with the time profile and phase plane pictures for this data ($400 \leq t \leq 500$). Do you think the system had settled down to its long term behavior by t=400? By t=500?

7. Relax one of the error tolerances in an ODESET for ode45 until the error is visually noticeable. Plot on the same graph the time profile of θ_1 for the visually accurate solution and the profile with graphically visible errors. Report what error tolerances you used.