Math 329 Unit 6 Fall 2014

Due: Friday, November 21

1. Examine the graphical agreement of using shooting and finite difference (using 100 points) to solve: $y'' + 2y' + 3y = 0$, with $y(0) = 1$, and $y(10) = 2$. Hand in a graph of the two solutions on the same plot and a sentence describing the difference. Use the scripts bvpshoot.m and bvpfd.m from our website.

   In a paragraph or two, describe two ways to control the error for shooting and one for finite difference. Then describe how you would decrease each error using your Matlab code.

2. Now solve the following equation using both methods.

$$y'' - 5y' - 50y = 0, \quad y(0) = 1, \quad y(10) = 1$$

   Use subplot to draw shooting in the top and relaxation in the bottom subplot. Adapt the bvpfd.m code as necessary (and include in your report). You shouldn't have to change the bvpshoot.m code. This problem is supposed to be hard for shooting to solve because the solution has a growing exponential solution in each direction in space. Report the error for the boundary conditions with each method. Do the boundaries of the solutions agree? Where do the solutions disagree?

3. Use the finite difference routine to solve: $y'' - \sin(x)y' + (x - 5)y = 0$ with boundary conditions $y(0) = 1$, and $y(10) = 1$ and hand in a graph of the solution with different grid spacings. Each curve should be a solution with points identified by symbols that are connected with lines (use formatting like'-o'). That way you can tell the grid size and the solution fairly easily. Use 10, 50 and 100 points. Put all three plots in their own subplot on a single graph.

4. Solve the following initial boundary value problem (PDE):

$$u_t = D * u_{xx} + u - u^3, \quad u(0, x) = 0.5\sin(x), \quad u_x(t, 0) = u_x(t, L) = 0$$

   for $D = 0.0001$. Use Crank-Nicholson's method in time for the derivatives and 3 point differences for spatial derivatives. Use a timestep of 0.01 for the timespan $[0, 6]$. The code appears in RD.m and stepRD.m (RD for reaction diffusion). Watch the animation. On the same graph, use subplot to draw two plots. In the top draw the solution versus time at the point x=0.32. In the bottom, draw the solution versus space for each half time unit $t = 0, 0.5, ..., 6$ (See the comments at the end of RD.m)

   What line number in the code of which file would you change to make: a) $u(0, x) = x - x^3$, b) a ten times finer grid, c) the pde $u_t = D * u_{xx} + (exp(u) - 1)/u$?