SECTION ONE: Multiple Choice (5 points for each answer: 50 points total)

1. On the planet Dangerboy, Captain Bananahands has stumbled across what appears to be a 3-sided coin. Let's call the sides *heads, tails,* and *feet.* Bananahands flips the coin 12 times and gets 6 feet. Assuming the coin is fair, what is the probability of this happening?

   A. .500 B. .111 C. .667 D. .392

   B *The experiment is a Binomial process. Hence, the answer is* $\binom{12}{6}(1/3)^6(2/3)^6 \approx .111$.

2. In a magical, far-away land, I found a 5-sided die. On Earth, a 5-sided die is necessarily not fair. I decide to test this newly found die for fairness by rolling it 400 times and counting the number of $\boxdot$'s. I counted 52.

   i) Assuming the die *is* fair, the number of $\boxdot$'s rolled is ______, give or take ______, or so.

   A. 80; 8 B. 80; 16 C. 200; 8 D. 200; 16

   A *Our distribution here is binary: 1,0,0,0,0. We expect (400)(1/5)=80* $\boxdot$ *'s (this is* EV_{sum}*). The second blank is* $SE_{sum} = \sqrt{400}\sqrt{(1/5)(4/5)} = 8$.

   ii) Assuming the die *is* fair, the chance of rolling 52 or fewer $\boxdot$'s is

   A) $\approx 0\%$ B) .03% C) .5% D) 99.997%

   B *We expecet* $400(1/5) = 80$ *with* $SE_{sum} = \sqrt{400}\sqrt{(1/5)(4/5)} = 8$. *Hence, using the Central Limit Theorem, our approximate probability is the area to the left of 52.5 (using the* .5 *correction for 0-1 boxes) on a Normal curve with mean 80 and standard deviation 8. We convert:* $\frac{52.5-80}{8} \approx -3.44$. *The corresponding area is .03%.*

3. A strange lottery at Britneyland Casinos is played as follows. The numbers 2, 3, 5, 7, and 13 are placed in a hat. The chairman of the lottery then picks at random 120 times (with replacement) from the hat. There are two types of winning "hands":

   (A) The number 5 is picked more than 35 times;

   (B) The sum of the picks is less than 500.

   Here are the questions:

   i) The number of fives picked is ______, give or take ______, or so.

   A. 24;.037 B. 24;3.65 C. 24; 4.38 D. 24; 8.76

   C *The distribution is binary: 0,0,1,0,0. Hence, the expected number of ones is* $120(1/5) = 24$ *with* $SE_{sum} = \sqrt{120}\sqrt{(1/5)(4/5)} \approx 4.38$.

   ii) The sum of the picks is ______, give or take ______, or so.

   A. 720; 42.7 B. 600; 36.8 C. 720; 36.8 D. 600; 42.7

   A *We use the distribution:* $2, 3, 5, 7, 13$ *which has an average of 6 and a standard deviation of 3.9. Hence, our expected sum is* $120(6) = 720$ *with* $SE_{sum} = \sqrt{120}(3.9) \approx 42.7$.

4. Someone rolls 100 four-sided dice. The sides are numbered 1 through 4 and are all equally likely to be rolled.

   i) The sum of the rolls should be ______, give or take ______, or so.

A. 150; 11   B. 150; 52   C. 250; 11   D. 250; 52

C *The first blank is the EV and the second blank is the SE for the sum. We have* $EV = 100\left(\frac{1+2+3+4}{4}\right) = 250$ *and* $SE = \sqrt{100}\sqrt{\frac{(1-2.5)^2+(2-2.5)^2+(3-2.5)^2+(4-2.5)^2}{4}} = 11.18$.

ii) What is the approximate chance that the sum of rolls is less than 300?

A. 15%   B. 75%   C. 95%   D. 99.9%

D *We use the information from part (a) to get the z-score of 300:* $z = \frac{300-250}{11.18} \approx 4.5$. *Consulting the Normal table for our approximation, we want the area to the left of 4.5. Clearly D is the closest answer.*

iii) The number of threes rolled should be ______, give or take ______, or so.

A. 15; 2   B. 15; 4   C. 25; 2   D. 25; 4

D *Here we have the binary distribution:* $0, 0, 1, 0$. *We find the EV and SE for sums. We have* $EV = 100(1/4) = 25$ *and* $SE = \sqrt{100}\sqrt{(1/4)(3/4)} \approx 4.33$.

iv) Estimate the chance that the number of threes rolled is between 15 and 20, exlusive.

A. 1%   B. 4%   C. 9%   D. 15%

C *We use the information from part (c) to convert 15.5 and 19.5 to standard units: 15.5 becomes* $\frac{15.5-25}{4.33} = -2.19$ *and* 19.5 *becomes* $\frac{19.5-25}{4.33} \approx -1.27$. *Consulting the normal table for our approximation, we want the area between* -2.19 *and* -1.27 *which is approximately* 8.8%.

SECTION TWO: Short Answer (5 points for each answer: 30 points total)

5. Regarding question #3 in the last section at Britneyland Casinos, which hand do you have a better chance of winning, (A) or (B)? Show all of your reasoning.

   (A) *For both we use the Central Limit Theorem to approximate the probabilities. For (A), we want* ≥ 36, *so we want the area to the right of 35.5 on a Normal curve with mean 24 and SD 4.38 (these numbers are from the solutions to question #4). Converting 35.5 we get a z-score of* $\frac{35.5-24}{4.38} \approx 2.63$. *Doing the same for (B), we want the area to the left of 500 on a Normal curve with mean 720 and SD 42.7. We get a z-score of* $\frac{500-720}{42.7} \approx -5.2$. *Since (B)'s z-score is farther out in the tail, (A) must give a larger area and hence a higher probability of occuring.*

6. A box contains 5 letters: A, B, C, D, E from which you are to draw 900 times, with replacement and at random.

   a) Write an expression (I do not want a numerical answer) for the *exact* probability that you draw between 180 and 200 A's.

   This is a binomial process with $n = 900$, $k = 180, 181, \ldots, 200$, *and* $p = 1/5$ *(the probability of picking an A). Hence, the expression is*

$$\binom{900}{180}(1/5)^{180}(4/5)^{720} + \binom{900}{181}(1/5)^{181}(4/5)^{719} + \cdots + \binom{900}{200}(1/5)^{200}(4/5)^{700}.$$

b) Approximate this probability (I want a numerical answer) using the Central Limit Theorem.

We are drawing 900 times from the box $1, 0, 0, 0, 0$. *This has* $EV_{sum} = 180$ *and* $SE_{sum} = \sqrt{900}\sqrt{(1/5)(4/5)} = 12$. *We want the area between 180 and 200, inclusive. So, converting* 179.5 *and* 200 *to z-scores, we want the area under the standard Normal curve between* $-.04 = \frac{179.5-180}{12}$ *and* $1.7 = \frac{200.5-180}{12}$. *This gives us approximately 47.5%, or .475 as a probability.*

7. TRUE or FALSE. For each question, circle true or false and give a reason to back up your answer. An answer alone is worth only 2 points if correct.

   i) You are more likely to get exactly 500 heads out of 1000 flips of a fair coin that you are to get 5 heads out of 10 flips with the same coin.

   TRUE FALSE (circle one) Reason:

   FALSE. Consider the SE_{sum} *as the number of flips increases. Since* $SE_{sum} = \sqrt{n}SD$, *as* n *increases so does the* SE. *Hence, the distribution of sums is getting more spread out. Thus, it is harder to get exactly some specified number with larger* n.

   ii) The Central Limit Theorem applies only to 0-1 Boxes.

   TRUE FALSE (circle one) Reason:

   FALSE. It applies to any box, provided the number of draws is large enough.

   iii) If 2 events E and F are mutually exclusive, then $P(E \text{ or } F) = P(E) + P(F)$.

   TRUE FALSE (circle one) Reason:

   TRUE. Mutually exlusive events cannot occur at the same time, so $P(E \text{ and } F) = 0$. *Using this in the addition rule gives the stated formula.*