Chapter 2 Practice Problems Solutions

Math 316
Robertson

1. A bowl has six white and five black balls. Reach in a (simultaneously) pick three balls at random. Find the probability of picking one white and two black balls.

$$\frac{\binom{6}{1}\binom{5}{2}}{\binom{11}{3}}$$

2. A committee of five is to be selected from six men and nine women. The selection is made randomly. What is the probability that the selected committed has exactly three men?

$$\frac{\binom{6}{3}\binom{9}{2}}{\binom{15}{5}}$$

3. The rules of poker are: You are dealt a five hand card from a deck of 52 distinct cards. There are four suits and 13 numbers in each suit (2,3,4,. . . ,10,J,Q,K,A). Find the following probabilities.

a) The probability of getting a straight flush, where a straight is a hand of consecutive values (Ace may be either high or low, so A,2,3,4,5 and 10,J,Q,K,A are both straights) and a flush means all cards are the same suit.

$$\frac{\binom{10}{1}\binom{4}{1}}{\binom{52}{5}}$$

b) The probability of getting a straight (where not all cards can be of the same suit).

$$\frac{\binom{10}{1}(4^5-4)}{\binom{52}{5}}$$

c) The probability of getting a flush (all cards are of the same suit, but the values of the cards does not form a straight).

$$\frac{\binom{4}{1}\left(\binom{13}{5}-10\right)}{\binom{52}{5}}$$

d) The probability of getting a full house, where a full house consists of three-of-a-kind *and* two-of-a-kind (i.e. a triple and a pair).

$$\frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}}$$

4. Roll three 6-sided differently colored dice, say blue, red and green. Find the probability that the value on the blue die is greater than or equal to the value on the red die and the value on the green die. Find the probability that the value on the blue die is greater than or equal to the value on the red die or the value on the green die. Finally, find the probability that the value on the blue die is greater than or equal to the value on the sum of the other two dice. (Hint: there are many separate cases to consider, but they all have easy probabilities to calculate. And remember, separate cases means addition.)

Let E be the event that Blue is $\geq$ Red;

let F be the event that Blue is $\geq$ Green.

For the first question, we want $P(E \cap F)$. Let's do this by cases. If blue is 1, then red and green must both be 1. This gives $\frac{1}{6} \cdot \left(\frac{1}{6}\right)^2$. If blue is 2, then red and green can each be 1 or 2. This gives $\frac{1}{6} \cdot \left(\frac{2}{6}\right)^2$. The full answer is:

$$\frac{1}{6} \cdot \left(\frac{1}{6}\right)^2 + \frac{1}{6} \cdot \left(\frac{2}{6}\right)^2 + \frac{1}{6} \cdot \left(\frac{3}{6}\right)^2 + \frac{1}{6} \cdot \left(\frac{4}{6}\right)^2 + \frac{1}{6} \cdot \left(\frac{5}{6}\right)^2 + \frac{1}{6} \cdot \left(\frac{6}{6}\right)^2 = \frac{91}{216}$$

For the second question we want $P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{7}{12} + \frac{7}{12} - \frac{91}{216} = \frac{161}{216}$. The $\frac{7}{12} = \frac{1}{6} + \frac{5}{12} = P(\text{same}) + P(\text{blue is higher})$.

5. You must flip one coin every second or the bomb under the bus will explode. What is the probability that you get exactly five heads in 10 seconds? What is the probability that you flip more heads than tails in 9 seconds? What is the probability that you flip more heads than tails in 10 seconds?

For 9 seconds, the probability is $\frac{1}{2}$. For 10 seconds, there is a chance of getting an equal number of heads and tails, i.e., 5 of each. The probability of getting exactly 5 heads out of 10 is

$$\frac{\binom{10}{5}}{2^10} = \frac{252}{2^{10}} = \frac{63}{256}.$$

Hence, we have a probability of $\frac{193}{216}$ that we have either more heads or more tails. Since these are equally likely events, the answer is $\frac{193}{512}$.

6. This is your first time playing Monopoly. You roll two dice, add their values, and move that many spaces. You notice that 7 spaces away is the first Chance space. What is the probability that you land on this space your first time around the board? (Hint: You don't necessarily have to land there on your first roll.)

We can roll a 7 or (2 and 5) or (3 and 4) or (2, 2, and 3).

$P(7) = 1/6$; $P(2,5) = 2 \cdot \frac{1}{36}\frac{4}{36} = \frac{1}{162}$; $P(3,4) = 2 \cdot \frac{2}{36}\frac{3}{36} = \frac{1}{108}$; $P(2,2,3) = 3 \cdot \frac{1}{36}\frac{1}{36}\frac{2}{36} = \frac{1}{7776}$. Summing over cases, we get $\frac{1417}{7776} \approx .182$.