Chapter 1 Homework Answers

Math 316
A. Robertson

p. 16: 1-22, 25, 27, 28.

1. a) By the multiplication principle we get $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 67,600,000$.

b) $26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 19,656,000$

2. $6^4 = 1296$

3. 20!

4. For the first question there are 4! arrangements. For the second question, Jay has two choices (piano, drums) and Jack is forced to the play the other. Then John has two choices (guitar, bass) and Jim is forced to play the other. Hence, there are only 4 arrangements.

5. There are $8 \cdot 2 \cdot 9 = 144$ possible codes. If they must start with 4 there are $1 \cdot 2 \cdot 9 = 18$ possible codes.

6. $7^4 = 2401$ kittens

7. a) $6! = 720$ (b) $2 \cdot 3! \cdot 3! = 72$ (c) $4! \cdot 3! = 144$ (d) $6 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 72$

Remarks: For (b) glue the girls together and glue the boys together. Then there are 2! ways to arrange the glued-together girls and the glued-together boys. Then there are 3! ways to arrange the boys, and 3! ways to arrange the girls. For (c), glue all of the boys together. There are 4! ways of arranging the girls and the glued-together boys. There are then 3! ways to arrange the boys. For (d) the arrangement is either BGBGBG or GBGBGB, hence we multiply by 2.

8. a) $5! = 120$ (b) $\frac{7!}{2!2!} = 1260$ (c) $\frac{11!}{4!4!2!} = 34,650$ (d) $\frac{7!}{2!2!} = 1260$

9. $\frac{12!}{6!4!} = 27,720$

10. a) $8! = 40320$ (b) $2 \cdot 7! = 10080$ (c) $2 \cdot (4!)^2 = 1152$ (d) $5! \cdot 4! = 2880$ (e) $4! \cdot 2^4 = 384$

Remarks: For (b) glue A and B together. There are 7! ways to arrange the 6 people and the glued-together group. Then there are 2! ways to arrange A and B. For (c) see comment 7d above. For (d) see comment 7c above. For (e), glue each couple together and arrange in 4! ways. Then for each couple there are 2!=2 arrangements.

11. a) 6! (b) $3! \cdot 2! \cdot 3!$ (c) $3! \cdot 4!$

12. a) 30^5 (b) $30 \cdot 29 \cdot 28 \cdot 27 \cdot 26$

13. $\binom{20}{2}$

14. $\binom{52}{5}$

15. There are $\binom{10}{5}\binom{12}{5}$ possible choices of the five men and five women. They can then be paired up in 5! ways. Hence, the answer is $5! \cdot \binom{10}{5}\binom{12}{5}$.

16. a) $\binom{6}{2}+\binom{7}{2}+\binom{4}{2} = 42$ possibilities. (b) $\binom{17}{2} - [\binom{6}{2}+\binom{7}{2}+\binom{4}{2}] = 94$.

17. The first gift can go to any of the 10 children, the second to the remaining 9, etc. The answer is $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 604,800$.

18. $\binom{5}{2}\binom{6}{2}\binom{4}{3} = 600$.

19. a) There are $\binom{8}{3}\binom{4}{3} + \binom{8}{3}\binom{2}{1}\binom{4}{2} = 896$ possible committees; there are $\binom{8}{3}\binom{4}{3}$ that do not contain either of the 2 men, and $\binom{8}{3}\binom{2}{1}\binom{4}{2}$ that contain exactly one of the men.

b) There are $\binom{6}{3}^2 + \binom{2}{1}\binom{6}{2}\binom{6}{3} = 1000$ possible committees.

c) There are $\binom{7}{3}\binom{5}{3} + \binom{7}{2}\binom{5}{3} + \binom{7}{3}\binom{5}{2} = 910$ committees; there are $\binom{7}{3}\binom{5}{3}$ in which neither feuding party serves, $\binom{7}{2}\binom{5}{3}$ in which the feuding woman serves, and $\binom{7}{3}\binom{5}{2}$ in which the feuding man serves.

20. a) $\binom{6}{5}+\binom{2}{1}\binom{6}{4}$ (b) $\binom{6}{5}+\binom{6}{3}$.

21. $\binom{7}{4} = 35$ paths since we must take 7 steps and from those choose 4 steps to go right.

22. We first got to the circled point and then from the circled point to the end. There are $\binom{4}{2} = 6$ paths to the circled point and there are $\binom{3}{2} = 3$ paths from the circled point to the end. Hence, the answer is $6 \cdot 3 = 18$.

25. There are $\binom{52}{13,13,13,13}$ hands.

27. $\binom{12}{3,4,5} = \frac{12!}{3!4!5!}$

28. a) 4^8 (b) $\binom{8}{2,2,2,2} = \frac{8!}{2^4} = 2520$.