Chapter 4 Homework Answers

p. 172: 1-10,17,20, 21, 23,28,37,38,41,51,52,54,58,61,63,71.

1. $X \in \{4, 2, 1, 0, -1, -2\}$: $P(X=-2) = \frac{\binom{8}{2}}{\binom{14}{2}} = 28/91$, $P(X=-1) = \frac{\binom{8}{1}\binom{2}{1}}{\binom{14}{2}} = 16/91$,
$P(X=1) = \frac{\binom{8}{1}\binom{4}{1}}{\binom{14}{2}} = 32/91$, $P(X=0) = \frac{\binom{2}{2}}{\binom{14}{2}} = 1/91$, $P(X=2) = \frac{\binom{4}{1}\binom{2}{1}}{\binom{14}{2}} = 8/91$, $P(X=4) = \frac{\binom{4}{2}}{\binom{14}{2}} = 6/91$

2. $X \in \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36\}$:$P(X=1) = 1/36, P(X=2) = 2/36, P(X=3) = 2/36, P(X=4) = 3/36, P(X=5) = 2/36, P(X=6) = 4/36, P(X=8) = 2/36, P(X=9) = 1/36, P(X=10) = 2/36, P(X=12) = 4/36, P(X=15) = 2/36, P(X=16) = 1/36, P(X=18) = 2/36, P(X=20) = 2/36, P(X=24) = 2/36, P(X=25) = 1/36, P(X=30) = 2/36, P(X=36) = 1/36$.

3. $X \in \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$:$P(X=3) = 1/216, P(X=4) = 3/216, P(X=5) = 6/216, P(X=6) = 10/216, P(X=7) = 15/216, P(X=8) = 21/216, P(X=9) = 25/216, P(X=10) = 27/216, P(X=11) = 27/216, P(X=12) = 25/216, P(X=13) = 21/216, P(X=14) = 15/216, P(X=15) = 10/216, P(X=16) = 6/216, P(X=17) = 3/216, P(X=18) = 1/216$.

4. First notice that $P(X=i) = 0$ for $i = 7, 8, 9, 10$. The others are: $P(X=1) = \frac{\binom{5}{1}9!}{10!} = 1/2$, $P(X=2) = \frac{\binom{5}{1}\binom{5}{1}8!}{10!} = 5/18$, $P(X=3) = \frac{\binom{5}{2}2!\binom{5}{1}7!}{10!} = 5/36$, $P(X=4) = \frac{\binom{5}{3}3!\binom{5}{1}6!}{10!} = 10/168$, $P(X=5) = \frac{\binom{5}{4}4!\binom{5}{1}5!}{10!} = 5/252$,
$P(X=6) = \frac{\binom{5}{5}5!\binom{5}{1}4!}{10!} = 1/252$

5. $X \in \{n, n-2, n-4, \ldots, 4-n, 2-n, -n\}$

6. $X \in \{3, 1, -1, -3\}$ $P(X=3) = P(X=-3) = 1/8; P(X=1) = P(X=-1) = 3/8$.

7. a) 1,2,3,4,5,6 b) 1,2,3,4,5,6 c) 2,3,. . . ,12 d) 5,4,3,. . . ,-3,-4,-5.

8. a) $P(X=i) = \frac{2i-1}{36}, i = 1, 2, 3, 4, 5, 6$ b) $P(X=i) = \frac{13-2i}{36}, i = 1, 2, 3, 4, 5, 6$

c) We know this already d) $P(X=i) = \frac{6-|i|}{36}, i = 5, 4, 3, \ldots, -3, -4, -5$

9. Here we can have a maximum occur once, twice, or even three times. Hence, $P(X=17) = \frac{\binom{3}{1}16^2}{20^3} + \frac{\binom{3}{2}16}{20^3} + \frac{\binom{3}{3}}{20^3}$.
However, since we are only interested in $X \geq 17$, we can do $P(X \geq 17) = 1 - P(X \leq 16) = 1 - \frac{16^3}{20^3}$

10. Let E_i denote the event that we win i dollars, and F the event that we win something. Then $P(E_i|F) = \frac{P(E_i \cap F)}{P(F)} = \frac{P(E_i)}{P(F)}$. From the example, $P(F) = 1/3$, hence $P(E_i|F) = 3P(E_i)$. The probabilities $P(E_i), i = 1, 2, 3$ are also given in the problem. The answers are $P(E_1|F) = 117/165, P(E_2|F) = 45/165, P(E_3|F) = 3/165$.

20. a) $P(X > 0) = P(win\ first\ spin) + P(lose\ first\ spin,\ win\ next\ two) = (18/38) + (20/38)(18/38)^2 = .5918$

b) No. Although there is a better that 50 % chance of winning, you only win \$1, whereas you may lose up to \$3 when you lose.

c) First we se that $P(X=-3) = (20/38)^3 = .1458$. Since $P(X=1) + P(X=-1) + P(X=-3) = 1$, we see that $P(X=-1) = .2624$. Thus, $E(X) = (\$1)(.5918) + (-\$1)(.2624) + (-\$3)(.1458) = -.108$.

21. a) $E(X) > E(Y)$ since there is a larger probability of choosing a student, say, from the bus with 40 than from the bus with 25.

b) $E(X) = 40(40/148) + 33(33/148) + 25(25/148) + 50(50/148) = 39.28$ and $E(Y) = (1/4)(40 + 33 + 25 + 50) = 37$

23. a) Use all of your money to buy 500 ounces of the commodity and then sell after one week. The expected amount of money you will get is $(1/2)(500) + (1/2)(2000) = 1250$

b) Do not immediately buy but use your money to buy after one week. The expected number of ounces of the commodity is $(1/2)1000 + (1/2)(250) = 625$.

28. Let X be the number of defective. Then $X \in \{0, 1, 2, 3, \}$: $P(X=0) = \frac{\binom{16}{3}}{\binom{20}{3}} = 560/1140$, $P(X=1) = \frac{\binom{4}{1}\binom{16}{2}}{\binom{20}{3}} = 480/1140$, $P(X=2) = \frac{\binom{4}{2}\binom{16}{1}}{\binom{20}{3}} = 96/1140$, $P(X=3) = \frac{\binom{4}{3}}{\binom{20}{3}} = 4/1140$.

Hence, $E(X) = 0(560/1140) + 1(480/1140) + 2(96/1140) + 3(4/1140) = 3/5$.

37. $Var(X) = E(X^2) - E^2(X) = E(X^2) - (39.28)^2$, where $E(X^2) = 40^2(40/148) + 33^2(33/148) + 25^2(25/148) + 50^2(50/148)$. Hence, $Var(X) = 82.2$. Similarly, $Var(Y) = (1/4)(40^2 + 33^2 + 25^2 + 50^2) - 37^2 = 84.5$.

38. $Var(X) = E(X^2) - E^2(X) = 5$. Thus, $E(X^2) = 6$.

a) $E((2+X)^2) = E(4 + 4X + X^2) = E(4) + 4E(X) + E(X^2) = 14$.

b) $Var(4 + 3X) = Var(3X) = 9Var(X) = 45$.

41. The pdf here is Binom$(10, 1/2)$. We want $P(X \geq 7) = \sum_{i=7}^{10} \binom{10}{i}(1/2)^i(1/2)^{10-i} = .172$.

51. This is Poisson(.2).

a) $P(X = 0) = e^{-.2} = .819$

b) $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - .819 - e^{-.2}(.2) = .017$

52. This is Poisson(3.5).

a) $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - e^{-3.5} - 3.5e^{-3.5} = .864$.

b) $P(X \leq 1) = P(X = 0) + P(X = 1) = 4.5e^{-3.5} = .136$.

54. Approximation, hence Poisson(2.2).

a) $P(X = 0) = e^{-2.2} = .111$

b) $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - 3.2e^{-2.2} = .645$

58. First, let me give the exact values:

a) $\binom{8}{2}(.1)^2(.9)^6 = .14880348$

b) $\binom{10}{9}(.95)^9(.05) = .3151247049$

c) $\binom{10}{0}(.1)^0(.9)^{10} = .3486784401$

d) $\binom{9}{4}(.2)^4(.8)^5 = .066060288$

And now the approximation by Poisson. We use $\lambda = np$ in each case.

a) $e^{-.8}\frac{.8^2}{2!} = .1437852685$ (good)

b) $e^{-9.5}\frac{9.5^9}{9!} = .1300025397$ (terrible)

c) $e^{-1}\frac{1^0}{0!} = .3678794412$ (not bad)

d) $e^{-1.8}\frac{1.8^4}{4!} = .07230173370$ (not bad)

61. Approximation, hence Poisson(1000(.0014)) = Poisson(1.4)

$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - 2.4e^{-1.4} = .4082$.

63. In 5 minutes we expect 2.5 people, on average, to enter. This is Poisson(2.5).

a) $P(X = 0) = e^{-2.5} = .0821$

b) $P(X \geq 4) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) = 1 - e^{-2.5} - 2.5e^{-2.5} - \frac{(2.5)^2}{2}e^{-2.5} - \frac{2.5^3}{3!}e^{-2.5} = .2424$.

71. a) $(26/38)^5 = .1500$

b) This is Geom(12/38). $P(X = 4) = (1 - 12/38)^3(12/38) = .1012$.