

Lab 3: Direction Fields and Euler's Method

In this lab we will use the Java versions of **dfield** and **pplane**, as well as Microsoft Excel, to investigate the behavior of selected differential equations. (Hint: Use Firefox, not Safari.)

U.S. Population Growth

The U.S. population was 250 million in 1990 and 304 million in 2008.

1. What was the natural growth rate of the US population during this period?
2. Assuming that the natural growth rate of the U.S. population remains constant, use dfield to predict the U.S. population in 2020, 2030, 2040, and 2050. In what year will the U.S. population reach 500 million?
3. Check your answers to the previous question by solving the initial value problem $\frac{dx}{dt} = rx$, $x(0) = 250$, where r is the natural growth rate.

China vs. India

China's population was 1.141 billion in 1990 and 1.333 billion in 2008, whereas India's population was 849 million in 1990 and 1.140 billion in 2008.

1. Calculate the relative growth rates for China and India during this period.
2. Use dfield to predict the population of both China and India in 2020, 2030, 2040, and 2050.
3. Which country will reach 2 billion first? In what year will India surpass China in population?

World Population

According to the **World Population Clock**, the current world population is 7.24 billion (as of 7/1/2014), and the current relative growth rate is 1.14%. In this section, we will try to model the world population using a logistic model $x' = rx(1 - x/M)$.

1. Use the chart "World Population: Past, Present, and Future" to estimate the world's human carrying capacity M .
2. Estimate the natural growth rate r using the the current population, the current relative growth rate, and your estimate for M .
3. Use dfield to plot the direction field for the logistic differential equation $x' = rx(1 - x/M)$ using your estimates for r and M . Click as close to (2014, 7.24) as possible to make dfield draw the solution passing through that point.

4. When will the world population reach 10 billion according to your model? According to the website?

Euler's Method

Now we are going to use Microsoft Excel to apply Euler's method to the law of natural growth, seasonal growth, and the predator-prey (Lotka-Volterra) equations.

The Law of Natural Growth

1. Assuming $x' = x$ and $x(0) = 1$, estimate $x(1)$ using 10, 50, 100, and 500 time steps.
2. What famous number are your estimates approaching?

Seasonal Growth

The growth rate of a species may be subject to seasonal variations caused by factors such as temperature, sunlight, or the availability of food. One way to model this situation is with the differential equation $x' = rx \cos(ct - d)$, where r , c , and d are parameters.

1. Assuming that $x(0) = 200$, $r = 0.1$, $c = 0.5236$, and $d = 2.618$, estimate $x(12)$ using 12, 48, and 360 time steps. In each case, make a scatter plot that shows x and x' as functions of t .
2. When does the population x reach its minimum and its maximum? What are the minimum and maximum populations?
3. When does the growth rate x' reach its minimum and maximum?
4. What is the period of the oscillation?
5. Now enter the same differential equation into dfield and compare its solution to what you found using Excel.

Predator-Prey

Now imagine that there are two species, one of which eats the other. This situation can be modeled by the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= rx - axy \\ \frac{dy}{dt} &= -sy + bxy\end{aligned}$$

where x is the number of prey, y is the number of predators, and a , b , r , and s are parameters.

1. Use Euler's method to simulate the number of predators and prey over time, using 200 time steps, each of size 1. Assume that $x(0) = 1000$, $y(0) = 50$, $a = 0.001$, $b = 0.00002$, $r = 0.08$, and $s = 0.02$.
2. What are the minimum and maximum values of x and y ?
3. Make a line graph showing x and y as functions of time.

4. Make a scatter plot of y versus x .
5. Repeat steps 1 through 4 with $x(0) = 1000$ and $y(0) = 60$.
6. Now enter the Lotka–Volterra equations into pplane. Click on the points $(1000, 50)$ and $(1000, 60)$. What do you observe?
7. For what number of prey and what number of predators will the system be in equilibrium?