CORRIGENDUM AND STATEMENT ON PRIORITY FOR: CONJUGACY CLASSES AND CLASS NUMBER

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The article contains some errors. The main theorem is not correct. The correct version is as follows:

Main Theorem There is a one-to-one correspondence between the conjugacy classes of matrices in $M_n(\mathbf{Z})$ having α as an eigenvalue – n being the degree of the minimal polynomial of α – and all ideal classes in the order $\mathbf{Z}[\alpha]$ (not just invertible ones).

A proof of this can be found in:

http://www.math.uconn.edu/~kconrad/blurbs/gradnumthy/matrixconj.pdf.

The basic idea is that fractional $\mathbf{Z}[\alpha]$-ideals (up to equivalence) are in one-to-one correspondence with $\mathbf{Z}[\alpha]$-modules (up to isomorphism) which are additively isomorphic to $\mathbf{Z}^n$. The latter is in one-to-one correspondence with the conjugacy classes under consideration. This is because giving a $\mathbf{Z}[\alpha]$-module structure is equivalent to giving a linear map $A : \mathbf{Z}^n \to \mathbf{Z}^n$ such that $m_\alpha(A) = 0$, where m_α is the minimal polynomial of α. Such module structures are isomorphic if and only if the corresponding matrices are conjugate to each other.

Also, this version of the theorem is quite old and can be found in “A Correspondence Between Classes of Ideals and Classes of Matrices” by C. Latimer and C. MacDuffee, Annals of Math., Vol. 34, No. 2, pp. $313 - 316$.

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