COPRIME MAPPINGS ON n-SETS

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Received: 11/22/16, Accepted: 10/19/17, Published: 11/10/17

Abstract

A bijection $f : A \to B$ on two sets of integers A and B is a *coprime mapping* if $\gcd(a, f(a)) = 1$ for all $a \in A$. Carl Pomerance and J. L. Selfridge proved that if n is a positive integer, $A = \{1, 2, \ldots, n\}$, and B is any set of n consecutive integers, then a coprime mapping from A onto B always exists. Here we consider coprime mappings on adjacent sets of n consecutive integers. We conjecture that if A is a set of n consecutive integers with $n \in A$ and B is the adjacent set of n consecutive integers, then a coprime mapping from A onto B always exists. We computationally verify that this conjecture holds for $n \leq 600$, and prove that it holds if $A = \{2, 3, 4, \ldots, n+1\}$ or if n or $n+1$ is prime.

1. Introduction

In 1963, Daykin and Baines [1] investigated the existence of coprime mappings between sets of consecutive integers. Let

$$A = \{s, s+1, \ldots, s+n-1\} \quad \text{and} \quad B = \{t, t+1, \ldots, t+n-1\}$$

be two sets of n consecutive integers with $s \leq t$. If $f : A \to B$ is a bijection such that $\gcd(a, f(a)) = 1$ for all $a \in A$, then f is called a *coprime mapping*. Daykin and

Baines' main result was that if $A = \{1, 2, \ldots, n\}$ and $B = \{n+1, n+2, \ldots, 2n\}$ then a coprime mapping from A onto B always exists. In 1980, Pomerance and Selfridge [2] proved the following more general result:

Theorem 1. *If n is a positive integer, $A = \{1, 2, 3, \ldots, n\}$, and $B = \{k, k+1, \ldots, k+n-1\}$ is any set of n consecutive integers, then a coprime mapping $f : A \to B$ exists.*

This theorem settled a conjecture of D. J. Newman. More recently, Robertson and Small [3] determined when a coprime mapping exists from $A = \{1, 2, 3, \ldots, n\}$ or $A = \{1, 3, 5, \ldots, 2n-1\}$ to a set of n integers in arithmetic progression. Note that in all of these results, the set A is either the first n integers or the first n odd integers, so in particular $1 \in A$ in all cases. Here we consider the existence of coprime mappings from a set A of n consecutive integers with $1 \notin A$.

We refer to a set of n consecutive integers as an *n-set* and call two n-sets A and B *adjacent* if the smallest integer in B is one more than the largest integer in A. In this paper we consider coprime mappings on adjacent n-sets A and B where $1 \notin A$, so we take

$$A = \{s, s+1, \ldots, s+n-1\} \text{ and } B = \{s+n, s+n+1, \ldots, s+2n-1\},$$

for some $s \geq 2$.

In general, the existence of a coprime mapping between sets A and B as above is not guaranteed. For the simplest example, observe that if $A = \{2, 3, 4\}$ and $B = \{5, 6, 7\}$ then no coprime mapping $f : A \to B$ exists since 6 shares a common divisor with every element in A. In fact, when $n = 3$ there are infinitely many adjacent 3-sets A and B such that there is no coprime mapping from A onto B. Indeed, there is no such map whenever 2 divides s and 3 divides $s+1$ because then $s+4$ will be divisible by 6 and will share a common divisor with every element of A. More generally, for each $n > 1$, Daykin and Baines showed there are infinitely many adjacent n-sets A and B with $1 \notin A$ for which there is no coprime mapping from A onto B by constructing an infinite family of examples where the largest integer in B shares a common divisor with every integer in A. Other types of examples are also possible. For instance, there is no coprime mapping from $A = \{9, 10, 11, 12\}$ onto $B = \{13, 14, 15, 16\}$ because 10 and 12 are both coprime to 13 and to no other element of B. Similarly, there is no coprime mapping from $A = \{66, 67, 68, 69, 70, 71\}$ onto $B = \{72, 73, 74, 75, 76, 77\}$ because 66 and 70 are both coprime to 73 and to no other element of B. Note that in all of these examples, except for $A = \{2, 3, 4\}$ and $B = \{5, 6, 7\}$, the cardinality of the sets A and B is smaller than the magnitude of the integers in these sets. It seems plausible that if $n \neq 3$ a coprime mapping always exists between consecutive n-sets if the magnitude of the integers in the sets is roughly n. Investigating this idea further led us to make the following conjecture:

Conjecture 1. Let n be a positive integer, $n \neq 3$, and A and B be adjacent n-sets with $n \in A$. Then a coprime mapping $f : A \to B$ exists.

In Section 2, we show that Conjecture 1 holds in three special cases: when $A = \{2, 3, 4, \ldots, n+1\}$, when n is prime, and when $n+1$ is prime. In Section 3, we computationally show that Conjecture 1 holds for $n \leq 600$. In Section 4, we briefly investigate Conjecture 1 in a different way by considering the following problem: for a fixed n, find the smallest value of s such that no coprime mapping exists from $A = \{s, s+1, \ldots, s+n-1\}$ to the adjacent n-set B. We solve this problem for $2 \leq n \leq 17$. All computations were done using SageMath [4].

2. Main Results

In this section we prove that Conjecture 1 holds in three special cases: when $A = \{2, 3, 4, \ldots, n+1\}$, when n is prime, and when $n+1$ is prime.

We begin with a lemma on coprime mappings from a set of integers in arithmetic progression to itself. This lemma will be used in our proof of Conjecture 1 in the case where $A = \{2, 3, 4, \ldots, n+1\}$.

Lemma 1. *Let b, n, and s be positive integers with $n \geq 2$, and $A = \{s+bt \mid 0 \leq t \leq n-1\}$. Then there exists a coprime mapping $f : A \to A$ if and only if $\gcd(s, b) = 1$ and s is odd if n is odd.*

Proof. If $\gcd(s, b) \neq 1$, then $\gcd(s, b)$ divides every element of A, so a coprime mapping $f : A \to A$ does not exist. So from now on assume $\gcd(s, b) = 1$. We separately consider n even and n odd.

If n is odd and s is even, then A consists of $(n+1)/2$ even integers and only $(n-1)/2$ odd integers. Since a coprime mapping $f : A \to A$ must map even integers to odd integers, such a mapping cannot exist in this case.

If n is odd and s is odd, then A has odd cardinality and consists of one more odd than even integer. Define $f : A \to A$ by

$$f(s+bt) = \begin{cases} s+b(t+1) & \text{if } t = 0 \text{ or } t \equiv 1 \bmod 2 \\ s+b(t-1) & \text{if } t \neq 0, t \neq 2, \text{ and } t \equiv 0 \bmod 2 \\ s+b(t-2) & \text{if } t = 2. \end{cases}$$

Note that $\gcd(s+bt, s+b(t \pm 1)) = \gcd(s+bt, b) = \gcd(s, b) = 1$ for all t. Also, since s is odd, $\gcd(s+2b, s) = \gcd(2b, s) = \gcd(b, s) = 1$. Therefore, $f : A \to A$ is a coprime mapping.

If n is even, then A contains the same number of odd and even integers. In this case, define $f : A \to A$ by

$$f(s+bt) = \begin{cases} s+b(t+1) & \text{if } t \equiv 0 \bmod 2 \\ s+b(t-1) & \text{if } t \equiv 1 \bmod 2. \end{cases}$$

The same argument as above shows that f is a coprime mapping. □

We now return to Conjecture 1. Note that it trivially holds when $n = 1$ or $n = 2$. We exclude $n = 3$ since no coprime mapping exists from $A = \{2, 3, 4\}$ onto $B = \{5, 6, 7\}$. Thus from now on we only consider $n > 3$.

As mentioned Section 1, Conjecture 1 holds when $A = \{1, 2, 3, \ldots, n\}$ by Daykin and Baines [1], and later by the more general result of Pomerance and Selfridge [2] stated in our Theorem 1. We next use Lemma 1 to prove Conjecture 1 in the case where $A = \{2, 3, \ldots, n+1\}$. Note that the lemma applies when A is an n-set (take $b = 1$).

Theorem 2. *Let $n > 3$. If $A = \{2, 3, \ldots, n+1\}$ and $B = \{n+2, n+3, \ldots, 2n+1\}$, then there exists a coprime mapping $f : A \to B$.*

Proof. Let A and B be as in the statement of the theorem. Following Daykin and Baines, we denote the sets of even integers in A and B by A_e and B_e respectively, and the sets of odd integers by A_o and B_o respectively. We prove that a coprime mapping $f : A \to B$ exists by showing that there is a coprime mapping from A_e onto B_o and a second coprime mapping from A_o onto B_e.

First consider mapping A_e onto B_o. Let $A' = \{1, 2, \ldots, n+1\}$ and $B' = \{n+2, n+3, \ldots, 2n+2\}$ be consecutive $(n+1)$-sets. Then A and A' contain the same even integers and B and B' contain the same odd integers. Also, by Theorem 1, there exists a coprime mapping $g : A' \to B'$. Since g must map even integers to odd integers, g restricted to A_e is a coprime mapping from A_e onto B_o as needed.

To show that there is a coprime mapping from A_o onto B_e, we first consider the case where no element of B_e is a power of 2. For $x \in B_e$ we have $n+2 \leq x \leq 2n$, so $2 < x/2 \leq n$. Thus $x/2 \in A$. Moreover, since x is not a power of 2 in this case, we can factor x as $x = 2^r \cdot l$ where l is odd, $l \neq 1$. Thus $l \in A_o$. We can therefore define a function $j : B_e \to A_o$ by $j(x) = l$ where $x = 2^r \cdot l$ and l is odd. To find a coprime mapping from A_o onto B_e we will show that j is a bijection and then use Lemma 1.

To see that $j : B_e \to A_o$ is a bijection, note that if $x \in B_e$ then $x > n$ and so $2x > 2n$. Thus $2x \notin B_e$ since $2n+1$ is the largest element of B. It follows that no two elements of B_e have the same largest odd divisor, and so $j : B_e \to A_o$ is injective. Thus j is a bijection since B_e and A_o have the same cardinality.

Now, by Lemma 1, there is a coprime mapping $h : A_o \to A_o$. Thus the composition function $f = j^{-1} \circ h$ is a coprime mapping from A_o onto B_e since j^{-1} simply multiplies an element of A_o by a power of two. Therefore a coprime mapping $f : A_o \to B_e$ exists in the case where B does not contain a power of two.

Finally, we consider a coprime mapping from A_o onto B_e in the case where B contains a power of two. In this case the set of odd integers l generated by factoring all $x \in B_e$ as above includes 1 and all but a single element of A_o. We denote the omitted element by a^* and let $A_o^* = (A_o - \{a^*\}) \cup \{1\}$. Then, as above, we have an injective function $j : B_e \to A_o^*$ defined by $j(x) = l$ where $x = 2^r \cdot l$ and l is odd. Again, by Lemma 1, there is a coprime mapping $h : A_o \to A_o$. Define $h^* : A_o \to A_o^*$ by

$$h^*(a) = \begin{cases} h(a)) & \text{if } h(a) \neq a^* \\ 1 & \text{if } h(a) = a^*. \end{cases}$$

Then the composition function $f = j^{-1} \circ h^*$ is a coprime mapping from A_o onto B_e as needed to complete the proof. $\square$

Note that, in contrast to Theorem 1, it is not the case that if $A = \{2, 3, \ldots, n+1\}$ and B is an n-set that is not adjacent to A then a coprime mapping $f : A \to B$ always exists, so Theorem 2 cannot be generalized in this way. For instance, let b be the product of all the primes that divide at least one element of A. Then if any multiple of b is an element of B a coprime mapping $f : A \to B$ cannot exist since b shares a common divisor with every element of A. Also, if n is odd and $B = A$ then no coprime mapping $f : A \to B$ exists by Lemma 1.

The next theorem proves that Conjecture 1 holds when n is prime.

Theorem 3. *Let $p > 3$ be a prime, and A and B be adjacent p-sets with $p \in A$. Then a coprime mapping $f : A \to B$ exists.*

Proof. Let p, A, and B be as in the statement of Theorem 3. To prove the theorem we consider $p + 2 \in A$ and $p + 2 \notin A$ separately, and in both cases give a coprime mapping $f : A \to B$.

If $p + 2 \in A$, define $f : A \to B$ by

$$f(k) = \begin{cases} 2p + 2 & \text{if } k = p \\ 2p & \text{if } k = p + 2 \\ k + p & \text{otherwise.} \end{cases}$$

This map is surjective since $p, p + 2 \in A$ and $k \in A$ if and only if $k + p \in B$. It is a coprime mapping since $p \neq 2$ implies $\gcd(p, 2p + 2) = \gcd(p, 2) = 1$ and $\gcd(p+2, 2p) = \gcd(p+2, p) = \gcd(2, p) = 1$. Also, $p \in A$ implies no other multiple of p is an element of A since A has cardinality p. Thus, if $k \in A$, $k \neq p$, and $k \neq p + 2$, then $\gcd(k, k + p) = \gcd(k, p) = 1$, as needed.

If $p+2 \notin A$ then $p-2 \in A$, since $p \in A$ and A has cardinality $p > 3$. In this case define $f : A \to B$ by

$$f(k) = \begin{cases} 2p-2 & \text{if } k = p \\ 2p & \text{if } k = p-2 \\ k+p & \text{otherwise.} \end{cases}$$

The same proof as above with appropriate sign changes shows that this is a coprime mapping. $\square$

We use a similar argument to prove that Conjecture 1 also holds when $n+1$ is prime.

Theorem 4. *Let $n > 3$ be a positive integer such that $n+1$ is prime. Let A and B be adjacent n-sets with $n \in A$. Then a coprime mapping $f : A \to B$ exists.*

Proof. Let $n = p-1$ for some prime $p \geq 5$, and A and B be adjacent n-sets with $n \in A$. Then $k \in A$ if and only if $k+p-1 \in B$. Let s and $\ell = s+p-2$ be the smallest and largest elements of A respectively. Take $s \geq 3$ since the theorem holds when $s = 1$ by Daykin and Baines [1] and $s = 2$ by Theorem 2. Thus $\ell \geq p+1$.

If $s \neq n$, then since $n \geq 4$ we have $p-2, p-1, p, p+1 \in A$ and $2p-3, 2p-2, 2p-1, 2p \in B$. Define $f : A \to B$ by

$$f(k) = \begin{cases} 2p & \text{if } k = p-2 \\ 2p-2 & \text{if } k = p \\ \ell+1 & \text{if } k = \ell \\ k+p & \text{otherwise.} \end{cases}$$

If $s = n$, then $\ell = 2n-1$ and $p, p+2 \in A$. If $p > 5$ then $2p, 2p+2 \in B$, and we define $g : A \to B$ by

$$g(k) = \begin{cases} 2p+2 & \text{if } k = p \\ 2p & \text{if } k = p+2 \\ \ell+1 & \text{if } k = \ell \\ k+p & \text{otherwise.} \end{cases}$$

If $p = 5$ then $h : A \to B$ given by $h(4) = 9$, $h(5) = 8$, $h(6) = 11$, $h(7) = 10$ is a coprime mapping.

We leave it to the reader to verify that the maps f, g, and h given above are indeed coprime mappings. $\square$

3. Computations

In this section we verify Conjecture 1 for $n \leq 600$ using an algorithm based on an idea of Pomerance and Selfridge in [2].

Pomerance and Selfridge's paper, *Proof of D. J. Newman's coprime mapping conjecture* [2], is devoted to a proof of the result stated as our Theorem 1. In the second section of the paper, Pomerance and Selfridge inductively describe their algorithm for the construction of the desired coprime mapping, and in the next five sections they provide their proof that the algorithm is always successful. Of special interest to us, however, is their simpler algorithm for a coprime mapping given at the end of the second section, even though they do not have a proof that this simpler algorithm is always successful. It is the main idea behind this simpler algorithm that we use in our algorithm for the construction of a coprime mapping between two adjacent n-sets A and B with $n \in A$. We use our algorithm to verify Conjecture 1 for $n \leq 600$, but similarly do not have a proof that the algorithm is successful for all n.

The main idea behind Pomerance and Selfridge's simpler algorithm for constructing a coprime mapping f from $A = \{1, 2, \ldots, n\}$ to any n-set B is to first find the image under f of those integers in A that are coprime to the fewest number of integers in B and then continue to those integers in A that are coprime to increasingly more integers in B. Let ϕ denote Euler's function and relabel the integers in A as $a_1, a_2, \ldots, a_n$ where $\phi(a_i)/a_i \leq \phi(a_{i+1})/a_{i+1}$ for $1 \leq i < n$. They begin by giving conditions for when $f(a_{n-1})$ is defined first. Then they inductively define f on the remaining integers in A by defining $f(a_i)$ to be the least integer in B coprime to a_i and not equal to $f(a_1), \ldots, f(a_{i-1})$ or $f(a_{n-1})$ if previously assigned.

In our algorithm for the construction of a coprime mapping between adjacent n-sets A and B with $n \in A$, we order *both* A and B in the manner suggested by Pomerance and Selfridge. Then we similarly run through the integers in A mapping each to the first integer in B with the opposite parity that it is coprime to and that has not yet been used as a value of f. We consider the last odd and even integers in A separately because occasionally they are not coprime to the remaining even and odd elements of B respectively. For instance, this occurs when a large prime is in A and an even multiple of that prime is in B and these are the final elements to be paired due to their high values of $\phi(k)/k$. Should this occur, our algorithm includes a simple fix of doing a swap with a previously matched pair. This step usually takes only a single iteration before a complete coprime mapping is found. Our algorithm is described below. Note that the algorithm begins with the smallest element of A being 3 because the cases where the smallest element of A is 1 or 2 are resolved in Theorem 1 and Theorem 2 stated above. The use of the algorithm and the fix is shown in the subsequent example.

Algorithm for Generating a Coprime Mapping Between n-Sets:

1. Input n.

2. Let $s := 3$.

3. Let A and B be adjacent n-sets such that the smallest element of A is s.

4. Let A_e, A_o, B_e, B_o be the sets of even and odd integers in A and B respectively. Note: The algorithm constructs a coprime mapping f from A onto B by first constructing f from A_e onto B_o, and then from A_o onto B_e by the same method.

5. Let m be the cardinality of A_e (which is the same as the cardinality of B_o). Relabel the elements of A_e as $a_1, a_2, \ldots, a_m$ and the elements of B_o as $b_1, b_2, \ldots, b_m$, where $\phi(a_i)/a_i \leq \phi(a_{i+1})/a_{i+1}$ and $\phi(b_i)/b_i \leq \phi(b_{i+1})/b_{i+1}$ for $1 \leq i < m - 1$.

6. Using the new ordering, begin the construction of f from A_e to B_o by successively mapping each of the first $m - 1$ integers in A_e to the first integer in B_o that it is coprime to and that has not yet been used as a value of f. That is, for $1 \leq i < m$, inductively define $f(a_i) := b_j$ if j is the smallest integer such that $\gcd(a_i, b_j) = 1$ and for $1 \leq k < j$ either $\gcd(a_i, b_k) \neq 1$ or $f(a_t) = b_k$ for some $1 \leq t < i$.

7. Let b_r be the remaining integer in B_o that has not been used as a value of f. If a_m and b_r are coprime then define $f(a_m) := b_r$, and the construction of f on A_e is complete. If a_m and b_r are not coprime then run back in reverse order through the pairs already matched until a pair $(a_t, f(a_t))$ is found such that a_m and $f(a_t)$ are coprime and a_t and b_r are coprime. Then perform a swap by defining $f(a_m) := f(a_t)$ and redefining $f(a_t)$ as $f(a_t) := b_r$. Again, the construction of f on A_e is complete.

8. Construct f from A_o to B_e by following Steps 5–7 for these sets. This completes the construction of the coprime mapping $f : A \to B$.

9. Repeat Steps 3–8 for $s := 4, 5, \ldots, n$.

To demonstrate how this algorithm works in practice, we provide an example where $n = 11$ and the smallest element of A is $s = 6$. Then $A = \{6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$, $B = \{17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27\}$, and the $\phi(k)/k$-ordered sets of even integers and odd integers are the following:

$$A_e = \{6, 12, 10, 14, 8, 16\}, \qquad A_o = \{15, 9, 7, 11, 13\},$$
$$B_o = \{21, 27, 25, 17, 19, 23\}, \qquad B_e = \{18, 24, 20, 22, 26\}.$$

To find a coprime mapping $f : A \to B$ the algorithm begins by constructing f from A_e onto B_o. In Step 6, it starts with $6 \in A_e$ and defines $f(6) := 25$ since 6 is not coprime to 21 or 27. Similarly 12 is not coprime to 21 or 27, and 25 is already a value of f, so $f(12) := 17$. Then $f(10) := 21$, $f(14) := 27$, and $f(8) := 19$. In

Step 7, the remaining integer in B_o is 23 and 16 is coprime to 23, so $f(16) := 23$, which completes the construction of f on A_e. To construct f from A_o onto B_e, the algorithm starts Step 6 with $15 \in A_o$ and defines $f(15) := 22$ since 15 is not coprime to 18, 24, or 20. Similarly, 9 is not coprime to 18 or 24, so $f(9) := 20$. The rest of the pairings in Step 6 are almost trivial because the remaining integers in A_o are prime, so $f(7) := 18$ and $f(11) := 24$. Now, in Step 7, the remaining integer in B_e is 26 and 13 and 26 are not coprime so the fix is needed. The first pair considered is suitable for a swap. This gives $f(13) := 24$ and $f(11)$ is redefined as $f(11) := 26$, which completes our construction of f on A_o. Thus we have a coprime mapping from A onto B.

For $n \leq 600$ we implemented our algorithm in SageMath [4]. We first precomputed $\phi(k)/k$ for $k \leq 600$ so that we could quickly reference these values without having to recompute the necessary values of $\phi(k)/k$ every time the program was run. We loaded the master list of pairs $(k, \phi(k)/k)$ into the program, then for each n and smallest integer $s \in A$, we used a function to generate the four sets A_e, A_o, B_e, B_o described in Step 4. The elements of these sets were stored as pairs of the form $(k, \phi(k)/k)$. We use SageMath's built-in sort function to sort the four sets of pairs in ascending order of the second component $\phi(k)/k$. Then a final function was used to follow Steps 6 and 7 of the algorithm and construct a coprime mapping $f : A \to B$. For small n where computing time was not an issue, the program saved the coprime mapping as a list and outputted it for review. For large n the size of the generated list was too unwieldy to review, so the output was changed to only report the success or failure of the construction of the coprime mapping (it was successful in every case we tried).

For all $n \leq 600$, the algorithm successfully constructed a coprime mapping between all n-sets A and B with $n \in A$. This verifies Conjecture 1 for these values and establishes the following theorem:

Theorem 5. *Let $4 \leq n \leq 600$ and A and B be adjacent n-sets with $n \in A$. Then a coprime mapping $f : A \to B$ exists.*

4. Adjacent n-Sets Without a Coprime Mapping

In this section we briefly consider the problem of determining when a coprime mapping between two adjacent n-sets does not exist.

In Section 1 we gave examples of adjacent n-sets A and B for which no coprime mapping exists from A onto B. If we could characterize all adjacent n-sets for which such a mapping does not exist, then we would also settle Conjecture 1. In fact, it would be sufficient to solve the following problem: for a fixed n, what is the smallest value of s, where $A = \{s, s+1, \ldots, s+n-1\}$, such that no coprime mapping exists from A to the adjacent n-set B. Here we computationally investigate this problem.

Let n and s be positive integers, $A = \{s, s+1, \ldots, s+n-1\}$, and $B = \{s+n, s+n+1, \ldots, s+2n-1\}$. For $2 \leq n \leq 17$, Table 1 shows the smallest value of s such that no coprime mapping exists from A onto B.

To do the computations for Table 1, we used SageMath [4] and modified the algorithm given in Section 3 to find coprime mapping between adjacent n-sets A and B for increasing values of $s > n$ (recall the algorithm restricts to $3 \leq s \leq n$) until a pair was found for which the algorithm failed. Then we verified that not only does the algorithm fail to construct a coprime mapping between this pair of adjacent n-sets A and B, but that no other coprime mapping can exist between them. Indeed, for $n \leq 9$ the algorithm failed because two elements in A or B are both coprime to the same element in the other set and to no other element in that set, a condition that prohibits the existence of any other coprime mapping between these sets. For $10 \leq n \leq 17$ the algorithm failed because there is an odd number in B that shares a common divisor with every even number in A, so again no other coprime mapping can exist. It is interesting to note that in every case this odd number was equal to the product of every prime less than or equal to $n+1$ (but note that the existence of this odd number in B does not guarantee that a coprime mapping cannot exist between the two sets because A could contain a power of 2, which could then be mapped to it). In both cases, we see that the numbers in sets A and B rapidly become much larger than n, further supporting Conjecture 1.

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n	Smallest Element of A	Smallest Element of B
2	3	5
3	2	5
4	9	13
5	9	14
6	66	72
7	65	72
8	50	58
9	51	60
10	1143	1153
11	1143	1154
12	14999	15011
13	14999	15012
14	14999	15013
15	14999	15014
16	255237	255253
17	255237	255254

Table 1: The smallest adjacent n-sets A and B without a coprime mapping