ON SOLUTIONS TO ERDŐS' LAST EQUATION

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Abstract

We focus on the solutions to Erdős's last equation $n(x_1+\cdots+x_n) = x_1\cdots x_n$. Shiu classified the solutions into the classes $C_\ell(n)$, where $\ell+1$ is the number of $x_i > 1$ and presented some sufficient conditions for $C_3(n) \neq \emptyset$. In this paper, we consider the set $C_3(n)$, which implies that we deal with the solutions to $n(n-4+a+b+c+d) = abcd$ with $a, b, c, d > 1$. We introduce a new method and prove that $C_3(n) \neq \emptyset$ if $n \not\equiv 1 \mod 180180$ and $n \geq 6$. Furthermore, we prove that $C_3(n) \neq \emptyset$ for any $6 \leq n \leq 10^{24}$.

1. Introduction

The equal-sum-and-product problem is known as finding the integer solutions to the Diophantine equation $x_1 + \cdots + x_n = x_1 \cdots x_n$. Considerable theoretical and computational research has been conducted for this problem [1, 2, 4, 5]. By contrast, some research has also been performed on the solutions of a similar equation:

$$n\sum_{i=1}^{n} x_i = \prod_{i=1}^{n} x_i, \tag{1}$$

where $n > 1$ and x_i are positive integers satisfying $x_1 \leq x_2 \leq \cdots \leq x_n$. As Erdős asked Richard Guy [3, Problem D24] if he knew anything about Equation (1), this equation is called Erdős's last equation (ELE_n).

We make several observations as the first step towards solving Equation (1). When $n = 2$, we can simply conclude that the only solutions (x_1, x_2) with $x_1 \leq x_2$ to the equation $2(x_1 + x_2) = x_1x_2$ are $(x_1, x_2) = (3, 6), (4, 4)$. Therefore, we assume $n \geq 3$ throughout this paper. In 2019, Shiu classified the solutions to ELE_n into sets as follows:

$$C_\ell(n) = \{(x_1, \ldots, x_n) \mid x_1 = \cdots = x_{n-\ell-1} = 1, 2 \leq x_{n-\ell} \leq \cdots \leq x_n\}.$$

Furthermore, he characterized the set $C_1(n)$ as

$$C_1(n) = \{(\underbrace{1, \ldots, 1}_{n-2}, n + a, n + b) \mid ab = 2n(n-1), 1 \leq a \leq b\}.$$

As $(\underbrace{1,\ldots,1}_{n-3},2,n,2n-1),(\underbrace{1,\ldots,1}_{n-3},3,n,n)\in C_2(n)$, we find that the set $C_2(n)$ is not empty for $n\geq 3$. By contrast, for $\ell\geq 3$, it is difficult to establish a deceptively simple result. Thus, for $\ell\geq 3$, Shiu introduced the following two conjectures.

Conjecture 1 (Conjecture $\mathcal{C}_\ell$ of [6]). Let ℓ be a positive integer with $\ell\geq 3$. There exists $n_0(\ell)$ such that $C_\ell(n)\neq\emptyset$ for $n\geq n_0(\ell)$.

Conjecture 2 (Hypothesis $\mathcal{H}_3$ of [6]). For $n\geq 6$, integers k and $a,b,c>1$ exist such that

$$n(n-4+a+b+c)\equiv 0 \mod k,\ n+k=abc,\ 1\leq k\leq n^2. \tag{2}$$

If Conjecture 2 for n is true, then $(1,\ldots,1,a,b,c,d)\in C_3(n)$ with $d=nS/k, S=n-4+a+b+c$, and so $C_3(n)\neq\emptyset$. Let $\Omega(n)$ be the number of prime divisors of n counted with multiplicity. Shiu indicated that if $\Omega(n+1)\geq 3$ or n is composite, Conjecture 2 is true. For the former case, we can take $k=1$ and $n+1=abc$. Moreover, for the latter we take $k=n=bc$ and $a=2$. As a generalization, Shiu proved the following result.

Theorem 1 (Shiu 2019 [6]). *Let $n\geq 6$. If n is composite, or if $n\not\equiv 1 \mod 1260$, then $C_3(n)\neq\emptyset$. Moreover, $C_3(n)\neq\emptyset$ for $6\leq n\leq 1260001$.*

In this paper, we extend Shiu's result for $C_3(n)\neq\emptyset$ as follows.

Theorem 2. *Let $n\geq 6$. Then, $C_3(n)\neq\emptyset$ if n satisfies at least one of the following three conditions:*

(i) n is composite, (ii) $n\not\equiv 1 \mod 180180$, (iii) $6\leq n\leq 10^{24}$.

The condition (iii) is checked via a computer search using Maxima. Moreover, we present at least one solution $(1,\ldots,1,x_{n-3},x_{n-2},x_{n-1},x_n)\in C_3(n)$ explicitly for each $6\leq n\leq 10^{24}$. To accomplish this, we create a table (see Appendix) to deal with primes $p\equiv 1 \mod 180180$. The table shows a list of sufficient conditions for $C_3(p)\neq\emptyset$ (together with the explicit solutions), which serve as a filter to sort for sorting out cases where $C_3(p)\neq\emptyset$, leaving only a handful of primes less than 10^{24} that must be dealt with directly (see Section 4).

2. Criteria Using the Omega Function

In this section, we show some lemmas that ensure $C_3(n)\neq\emptyset$ using the Omega function Ω. As Shiu showed that if $n\not\equiv 1 \mod 1260$ or n is composite then $C_3(n)\neq\emptyset$, it suffices to consider the case that n is a prime $p\equiv 1 \mod 1260$.

Lemma 1. *Let p be a prime with $p \equiv 1 \mod 1260$. If* $\max\{\Omega(p+1), \Omega(p+2), \Omega(p+4)\} \geq 3$*, then* $C_3(p) \neq \emptyset$.

Proof. When $\Omega(p+1) \geq 3$, Shiu demonstrated this statement [6]. Therefore, it suffices to show the case $\Omega(p+2) \geq 3$ or $\Omega(p+4) \geq 3$. First, we assume $\Omega(p+2) \geq 3$. Then there exists a triple of integers $(s, t, u) \in \mathbf{Z}^3_{\geq 2}$ such that $p + 2 = stu$. As $p \equiv 1 \mod 1260$, p is an odd prime and $p + 2, s, t$ and u are also odd integers. Therefore, $(n, a, b, c, k) = (p, s, t, u, 2)$ satisfies Congruence (2), and Hypothesis $\mathcal{H}_3$ holds for p with $\Omega(p + 2) \geq 3$.

Next, we prove that if $\Omega(p + 4) \geq 3$, then $C_3(p) \neq \emptyset$. As aforementioned, there exists a triple of odd integers $(s, t, u) \in \mathbf{Z}^3_{\geq 3}$ such that $p+4 = stu$. As $p \equiv 1 \mod 4$, one can check that $p - 4 + s + t + u \equiv 0 \mod 4$, and Hypothesis $\mathcal{H}_3$ holds for p with $\Omega(p + 4) \geq 3$, independent of the choice (s, t, u).

As Hypothesis $\mathcal{H}_3$ implies $C_3(p) \neq \emptyset$, this lemma holds. $\square$

Next, we consider $\Omega(p + i)$ for $i \in \{5, 6\}$. Let p be a prime with $\Omega(p + 5) = 3$ and $p \equiv 1 \mod 1260$. Then we find $p + 5 = 2 \cdot 3 \cdot q$ with a prime $q \equiv 1 \mod 5$. As $p - 4 + 2 + 3 + q \equiv 3 \mod 5$, there exists no 5-tuple $(p, a, b, c, 5)$ satisfying Congruence (2). Furthermore, if p be a prime with $p \equiv 1 \mod 6$ and $p+6 = 7 \cdot q \cdot r$ with primes $q \equiv r \equiv 5 \mod 6$, then $p - 4 + 7 + q + r \equiv 4 \mod 6$. Therefore, there exists no 5-tuple $(p, a, b, c, 6)$ satisfying Congruence (2). By contrast, the following lemma holds.

Lemma 2. *Let p be an odd prime with $p \equiv 1 \mod 1260$. If* $\max\{\Omega(p + 5), \Omega(p + 6)\} \geq 4$*, then* $C_3(p) \neq \emptyset$.

Proof. First, we assume $\Omega(p + 5) \geq 4$. Then there exists a pair of integers $(s, t) \in \mathbf{Z}^2_{\geq 2}$ such that $p+5 = 6st$. As $p \equiv 1 \mod 5$, $(s \mod 5, t \mod 5) \in \{(1, 1), (2, 3), (4, 4)\}$. (If necessary, we swap the role of s and t.) We can choose a 5-tuple $(p, a, b, c, 5)$ satisfying Congruence (2) as follows:

$$(a, b, c) = \begin{cases} (6, s, t) & \text{if } (s, t) = (1, 1), \\ (2, s, 3t) & \text{if } (s, t) = (2, 3), (4, 4). \end{cases}$$

Therefore, Hypothesis $\mathcal{H}_3$ holds for any prime p with $\Omega(p + 5) \geq 4$.

Next, we prove that if $\Omega(p + 6) \geq 4$, then $C_3(p) \neq \emptyset$. As above, there exists a triple of odd integers $(s, t, u) \in \mathbf{Z}^3_{\geq 3}$ such that $p + 6 = 7stu$. As $p \equiv 1 \mod 6$, it follows that $(s \mod 6, t \mod 6, u \mod 6) \in \{(1, 1, 1), (1, 5, 5)\}$ (if necessary, we swap the role of s, t and u). Then the 5-tuple $(p, 7, s, tu, 6)$ satisfies Congruence (2). Therefore, Hypothesis $\mathcal{H}_3$ holds for any prime p with $\Omega(p + 6) \geq 4$.

This proves the lemma. $\square$

Next, we deal with $\Omega(p + 3)$. If $\Omega(p + 3) = 4$ and $p + 3 = 4qr$ with $q \equiv r \equiv 2 \mod 3$, following which, $p-4+a+b+c \equiv 2 \mod 3$ for any triple (a, b, c) with $abc =$

$4qr$. Hence, there is no 5-tuple $(p, a, b, c, 3)$ satisfying Congruence (2). Similarly, if $\Omega(p + 9) = 4$ and $p + 9 = 10qr$ with $q \equiv r \equiv 2 \mod 3$, then there is no 5-tuple satisfying Congruence (2).

Lemma 3. *Let p be an odd prime with $p \equiv 1 \mod 1260$. If $\max\{\Omega(p + 3), \Omega(p + 9)\} \geq 5$, then $C_3(p) \neq \emptyset$.*

Proof. First, we assume $\Omega(p+3) \geq 5$. Then there exists a triple of integers $(s, t, u) \in \mathbf{Z}^3_{\geq 2}$ such that $p + 2 = 4stu$. As $p \equiv 1 \mod 3$, $(s \mod 3, t \mod 3, u \mod 3) \in \{(1, 1, 1), (2, 2, 1)\}$ (if necessary, we swap the role of s, t and u). Then the 5-tuple $(p, 4, st, u, 3)$ satisfies Congruence (2). Therefore, Hypothesis $\mathcal{H}_3$ holds for any prime p with $\Omega(p + 3) \geq 5$.

Additionally, if $\Omega(p + 9) \geq 5$ then $p + 9 = 10stu$. As $p \equiv 1 \mod 3$, $(s \mod 3, t \mod 3, u \mod 3) \in \{(1, 1, 1), (2, 2, 1)\}$ (if necessary, we swap the role of s, t and u). Then the 5-tuple $(p, 10, st, u, 9)$ satisfies Congruence (2). Therefore, Hypothesis $\mathcal{H}_3$ holds for any prime p with $\Omega(p + 9) \geq 5$.

This proves the lemma. □

Next, we compute a lower bound on $\Omega(p+7)$. If $\Omega(p+7) = 6$ and $p+7 = 4q_1q_2q_3q_4$ with $q_i \equiv 2 \mod 7$, then it holds that $p - 4 + a + b + c \not\equiv 0 \mod 7$ for any triple (a, b, c) with $abc = 4q_1q_2q_3q_4$. Hence, there is no 5-tuple $(p, a, b, c, 7)$ satisfying Congruence (2). By contrast, we can prove the following lemma.

Lemma 4. *Let p be an odd prime with $p \equiv 1 \mod 1260$. If $\Omega(p + 7) \geq 7$, then $C_3(p) \neq \emptyset$.*

Proof. If $\Omega(p+7) \geq 7$ and $p \equiv 1 \mod 1260$, then we can factor $p+7 = 4q_1q_2q_3q_4q_5$. One can check that when order is irrelevant, there are 42 possible quintuples $(r_1, \ldots, r_5) \in \{1, 2, 3, 4, 5, 6\}^5$ with $q_i \equiv r_i \mod 7$. Then we can choose a pair $(p, a, b, c, 7)$ satisfying Congruence (2) as follows:

$$(a, b, c) = \begin{cases} (q_i, q_j, \frac{p+7}{q_iq_j}) & \text{if } (r_i, r_j) \in \{(1, 1), (3, 3), (3, 4)\}, \\ (2q_i, 2q_j, \frac{p+7}{4q_iq_j}) & \text{if } (r_i, r_j) \in \{(2, 5), (3, 4), (4, 4), (5, 5)\}, \\ (q_i, 4q_j, \frac{p+7}{4q_iq_j}) & \text{if } r_i = 1, r_j = 2, \\ (4q_i, q_jq_k, \frac{p+7}{4q_iq_jq_k}) & \text{if } r_i = 2, r_j = r_k = 6, \\ (q_i, q_jq_k, \frac{p+7}{q_iq_jq_k}) & \text{if } r_i = 1, r_j = r_k = 6. \end{cases}$$

As any quintuple satisfies one of the above five conditions, the proof is complete. □

We can also compute the sufficient condition of $\Omega(p+i)$ for $C_3(p) \neq \emptyset$ for a larger integer i. For example, we can obtain lower bounds on $\Omega(p + 14)$ and $\Omega(p + 20)$ as follows.

Lemma 5. *Let p be an odd prime with $p \equiv 1 \mod 1260$. Then the following hold.*

1. *If* $\Omega(p+14) \geq 5$*, then* $C_3(p) \neq \emptyset$*.*

2. *If* $\Omega(p+20) \geq 4$*, then* $C_3(p) \neq \emptyset$*.*

Proof. First, we assume $\Omega(p+14) \geq 5$. Then we can factor $p+14 = 15stu$. Without loss of generality, we can assume $s \equiv 1, 3, 9, 11 \mod 14$ (if necessary, we swap the role of s, t and u). We can choose a 5-tuple $(p, a, b, c, 14)$ satisfying Congruence (2) as follows:

$$(a, b, c) = \begin{cases} (15, s, tu) & \text{if } s \equiv 1 \mod 14, \\ (3, s, 5tu) & \text{if } s \equiv 3, 11 \mod 14, \\ (3, 5s, tu) & \text{if } s \equiv 9 \mod 14. \end{cases}$$

Next, we prove the second assertion. If $\Omega(p+20) \geq 4$, we can factor $p+20 = 21st$. Without loss of generality, we can assume $s \equiv 1, 7, 9, 11, 17, 19 \mod 20$ (if necessary, we swap the role of s and t). Then, we can choose a 5-tuple $(p, a, b, c, 20)$ satisfying Congruence (2) as follows:

$$(a, b, c) = \begin{cases} (21, s, t) & \text{if } s \equiv 1, 11 \mod 20, \\ (7, s, 3t) & \text{if } s \equiv 7, 9, 17, 19 \mod 20. \end{cases}$$

Based on the above argument, the proof is complete. □

Furthermore, when k is not a divisor of 1260, it does not necessarily follow that $p \equiv 1 \mod k$, and the computation of lower bound on $\Omega(p+k)$ may be slightly complicated. The least positive integer that is not a divisor of 1260 is 8. To conclude this section, we prove the following lemma about the lower bound on $\Omega(p+8)$.

Lemma 6. *Let* p *be an odd prime with* $p \equiv 1 \mod 1260$*. If* $\Omega(p+8) \geq 5$*, then* $C_3(p) \neq \emptyset$*.*

Proof. Assuming $\Omega(p+8) \geq 5$, there exists a triple (s, t, u) with $p+8 = 9stu$. Without loss of generality, we can assume $s \equiv 1, 5 \mod 8$ (if necessary, we swap the role of s, t and u). We can check the 5-tuple $(p, 9, s, tu, 8)$ satisfying Congruence (2), independent of the remainder of p divided by 8.

Therefore, we have proven the lemma. □

3. Criteria on Modulus

In this section, we consider $C_3(p)$ for a prime $p \geq 7$. First, we show the following lemma to introduce a way different from Shiu's way.

Lemma 7. *Let* p *be a prime number. If* $(x_1, \ldots, x_p) \in C_3(p)$*, then* $p | x_p$*.*

Proof. Let $(x_1, \ldots, x_p) \in C_3(p)$. If we assume $x_{p-1} \geq p$, then we have

$$x_{p-3} = \frac{p(\sum_{i=1}^{p-4} x_i + x_{p-2} + x_{p-1} + x_p)}{\prod_{i=1}^{p-4} x_i x_{p-2} x_{p-1} x_p - p} \leq \frac{p(p-4+2+p+p)}{2p^2 - p} \leq \frac{3}{2}.$$

This contradicts $(x_1, \ldots, x_p) \in C_3(p)$, and it follows that $x_{p-1} < p$.

As it holds that

$$p \sum_{i=1}^{p} x_i = \prod_{i=1}^{p} x_i,$$

p divides $\prod_{i=1}^{p} x_i$. Combining this with $x_{p-1} < p$, it is proven that $p|x_p$. $\square$

Lemma 7 ensures that we can denote $x_p = kp$ with $k \geq 1$. In this case, Erdős's last equation becomes

$$p(p - 4 + x_{p-3} + x_{p-2} + x_{p-1} + kp) = x_{p-3} x_{p-2} x_{p-1} kp,$$

that is,

$$x_{p-3} = \frac{p - 4 + x_{p-2} + x_{p-1} + kp}{x_{p-2} x_{p-1} k - 1},$$

where $k \geq 1$ and $2 \leq x_i < p$ for $i \in \{p-3, p-2, p-1\}$. For simplicity, we forget the condition $x_{p-3} \leq x_{p-2} \leq x_{p-1}$. Therefore, we only need to find a triple of positive integers $(a, b) \in \mathbf{Z}^2_{\geq 2}, c \geq 1$ such that the congruence

$$p - 4 + a + b + cp \equiv 0 \mod abc - 1 \tag{3}$$

holds. We analyze whether $C_3(p)$ is empty for all primes in an arithmetic progression using the method of Shiu's paper. First, we prove the following theorem.

Theorem 3. *For any prime $p \not\equiv 1 \mod 11$, it holds that $C_3(p) \neq \emptyset$.*

Proof. It suffices to show the case $p \equiv 1 \mod 1260$. If $p \equiv 2 \mod 11$, then we can express $p = 13860m + 2521$. Assuming $k = 20$ in Congruence (2), we find $p + 20 = 231(60m + 11) = 3 \cdot 7 \cdot 11 \cdot (60m + 11)$ and $p - 4 + 11 + 21 + (60m + 11) \equiv 0 \mod 20$. Therefore, $(n, a, b, c, k) = (13860m + 2521, 11, 21, 60m + 11, 20)$ satisfies Congruence (2), and Hypothesis $\mathcal{H}_3$ holds for $p \equiv 2 \mod 11$. Next, we find triples (a, b, c) satisfying Congruence (3). By direct computation, we find the following table:

$p \mod 11$	(a, b, c)	$p \mod 11$	(a, b, c)
3	$(10, 10, 1)$	7	$(3, 2, 2)$
4	$(3, 4, 1)$	8	$(2, 4, 7)$
5	$(4, 7, 2)$	9	$(2, 6, 1)$
6	$(5, 9, 1)$	10	$(3, 5, 3)$

We can confirm that $p-4+a+b+cp > abc-1$ in each case. Thus, the p-tuple

$$\left(\underbrace{1,\ldots,1}_{p-4}, a, b, \frac{p-4+a+b+cp}{abc-1}, cp\right)$$

is an element of $C_3(p)$. This proves the assertion. □

This theorem provides an affirmative answer to the Shiu's question.

Question 1 (Shiu [6])**.** Is Hypothesis $\mathcal{H}_3$ true for all the primes in the arithmetic progression $n \equiv 13 \mod 88$?

Next, we obtain another sufficient condition for $C_3(p) \neq \emptyset$.

Theorem 4. *For* $p \not\equiv 1 \mod 13$*, it holds that* $C_3(p) \neq \emptyset$*.*

Proof. As Theorem 1 and Theorem 3 hold, it suffices to show the case $p \equiv 1 \mod 13860$. We seek a triple of positive integers (a,b,c) satisfying either Congruence (3) or Shiu's Congruence (2). Using direct computation, we find the following table for any prime $p = 180180m + 13860i + 1$ $(i = 1, 2, \ldots, 12)$:

$p \mod 13$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
2	$(5,21,1)$	
3	$(12,12,1)$	
4	$(2,7,1)$	
5		$(9,13,1540m+237)$
6	$(3,3,3)$	
7		$(7,13,1980m+457)$
8	$(10,4,1)$	
9	$(3,9,1)$	
10		$(4,13,3465m+2932)$
11	$(7,7,4)$	
12		$(2,13,6930m+6397)$

We can confirm that all triples provides a solution to the equation

$$p(p-4+x_{p-3}+x_{p-2}+x_{p-1}+x_p) = x_{p-3}x_{p-2}x_{p-1}x_p$$

with $x_i \geq 2$ $(i = p-3, p-2, p-1, p)$.

This proves the theorem. □

Combining Theorem 3 and Theorem 4 with Theorem 1, we deduce the following.

Theorem 5. *Let* $p \geq 7$ *be a prime. If* $p \not\equiv 1 \mod 180180$*, then it holds that* $C_3(p) \neq \emptyset$*.*

We can obtain similar results for other primes p theoretically. For example, if $p \not\equiv 1, 9 \mod 19$ then $C_3(p) \neq \emptyset$. Additionally, for any prime $q < 110$, we compute which of the positive integers $r < q$ satisfy the condition that if $p \equiv r \mod q$ then $C_3(p) \neq \emptyset$. We summarize our computation in the supplement table in Appendix.

4. Extension of the Interval with $C_3(n) \neq \emptyset$

In this section, we extend Shiu's result for $C_3(n) \neq \emptyset$. Shiu observed that $C_3(n)$ is not empty for $6 \leq n \leq 1260001$; refer to Theorem 1 [6]. In Sections 2 and 3, we presented additional sufficient conditions for $C_3(p) \neq \emptyset$. To simplify our computation, we demonstrate one more sufficient condition for $C_3(p) \neq \emptyset$.

Theorem 6. *If $p \equiv 96 \mod 323$, then $C_3(p) \neq \emptyset$.*

Proof. For any prime p with $p \equiv 96 \mod 323$, we find that $(a, b, c) = (4, 9, 9)$ satisfies Congruence (3). Thus,

$$\left(\underbrace{1, \ldots, 1}_{p-4}, 4, 9, \frac{10p+9}{323}, 9p\right) \in C_3(p).$$

$\square$

Applying the aforementioned sufficient conditions for $C_3(p) \neq \emptyset$ and the supplement table in Appendix, we find that $C_3(n) \neq \emptyset$ for any integer $6 \leq n \leq 10^{24}$, except for

$$\mathrm{Ex} = \left\{ \begin{array}{l} 1912923666245570738760 1, 705847842844354964754 01, \\ 2848314545752487049037 81, 6291049942453236076116 01, \\ 7770870228487766111712 01 \end{array} \right\}.$$

By contrast, we can find an element of $C_3(n)$ for each $n \in \mathrm{Ex}$ as shown in the following table:

n	(a, b, c) satisfying (3)
19129236662455707387601	$(8, 43, 27406921297953567 47815)$
70584784284435496475401	$(3, 882263, 5041772210894058262707)$
284831454575248704903781	$(274, 617, 1684815001805585686)$
629104994245323607611601	$(4, 197161, 2097019306499946899 06701)$
777087022848776611171201	$(8, 837439, 1110124484057660990 03029)$

Therefore, we have extended Shiu's result for $C_3(n) \neq \emptyset$ as follows.

Theorem 7. *It holds that $C_3(n) \neq \emptyset$ for $6 \leq n \leq 10^{24}$.*

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Appendix

In this appendix, we present a list of sufficient conditions for $C_3(p) \neq \emptyset$ combined with the explicit solutions. As shown in the previous sections, it suffices to find a triple (a, b, c) satisfying either our Congruence (3) or Shiu's Congruence (2).

For $p = 180180(17m + i) + 1$ $(i = 1, \dots, 16)$:

p mod 17	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2	$(4, 13, 1)$	$(4, 13, 58905m + 38116)$
3	$(5, 15, 5)$	$(3, 17, 60060m + 17665)$
4	$(2, 3, 3)$	$(2, 9, 170170m + 100101)$
5	$(2, 9, 1)$	$(3, 6, 170170m + 40041)$
6	$(3, 6, 1)$	$(9, 17, 20020m + 17665)$
7		$(9, 17, 20020m + 725)$
8		
9	$(3, 4, 10)$	$(3, 4, 255255m + 45055)$
10		$(4, 17, 45045m + 37096)$
11		
12		$(2, 17, 90090m + 10599)$
13	$(5, 7, 1)$	$(5, 17, 36036m + 27557)$
14		$(7, 17, 25740m + 10599)$
15	$(11, 31, 1)$	$(3, 17, 60060m + 3533)$
16	$(8, 15, 1)$	$(2, 17, 90090m + 63593)$

For $p = 180180(19m + i) + 1$ $(i = 1, \dots, 18)$:

p mod 19	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2	$(3, 15, 11)$	$(3, 15, 76076m + 52063)$
3	$(37, 37, 1)$	
4	$(11, 15, 3)$	$(11, 15, 20748m + 1095)$
5		$(3, 95, 12012m + 8851)$
6	$(4, 10, 10)$	$(2, 133, 12870m + 5419)$
7	$(4, 5, 1)$	$(13, 19, 13860m + 1459)$
8	$(9, 17, 1)$	
9		
10	$(6, 16, 1)$	$(10, 19, 18018m + 2845)$
11		$(2, 627, 2730m + 2299)$
12	$(7, 11, 1)$	$(7, 11, 44460m + 23401)$
13	$(2, 4, 12)$	$(7, 19, 25740m + 5419)$
14		$(2, 19, 90090m + 80607)$
15	$(2, 10, 1)$	$(5, 19, 36036m + 20863)$
16	$(3, 11, 15)$	$(4, 19, 45045m + 1459)$
17	$(2, 12, 4)$	$(3, 19, 60060m + 56899)$
18	$(2, 5, 2)$	$(2, 19, 90090m + 56899)$

For $p = 180180(23m + i) + 1$ $(i = 1, \dots, 22)$:

p mod 23	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2	$(4, 13, 4)$	$(23, 45, 4004m + 1915)$
3	$(4, 4, 13)$	$(7, 69, 8580m + 8207)$
4	$(4, 37, 7)$	
5	$(7, 10, 1)$	$(7, 10, 59202m + 54055)$
6		
7		
8	$(3, 8, 1)$	
9	$(2, 3, 4)$	$(2, 3, 690690m + 570574)$
10		
11	$(2, 4, 3)$	
12	$(3, 3, 18)$	$(12, 23, 15015m + 3917)$
13	$(3, 18, 3)$	
14	$(2, 6, 2)$	$(2, 6, 345345m + 75077)$
15	$(7, 37, 4)$	
16		$(4, 115, 9009m + 1567)$
17	$(5, 81, 5)$	
18	$(2, 18, 1)$	$(2, 69, 30030m + 3917)$
19	$(5, 5, 81)$	$(5, 23, 36036m + 21935)$
20	$(4, 6, 1)$	
21	$(4, 7, 37)$	$(3, 23, 60060m + 33947)$
22	$(3, 4, 2)$	$(2, 23, 90090m + 3917)$

For $p = 180180(29m + i) + 1$ $(i = 1, \ldots, 28)$:

p mod 29	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3	$(4, 17, 3)$	
4	$(3, 3, 13)$	$(3, 3, 580580m + 20033)$
5		
6		
7	$(4, 16, 5)$	
8	$(2, 15, 1)$	$(2, 15, 174174m + 72073)$
9		$(7, 87, 8580m + 6509)$
10	$(3, 10, 1)$	$(3, 39, 44660m + 4621)$
11	$(5, 6, 1)$	
12		
13		
14	$(5, 16, 4)$	
15	$(7, 25, 1)$	$(15, 29, 12012m + 9941)$
16		$(2, 29, 90090m + 15533)$
17		
18	$(3, 5, 2)$	$(3, 5, 348348m + 300302)$
19		
20	$(3, 17, 4)$	$(9, 13, 44660m + 24641)$
21	$(3, 5, 3)$	$(2, 63, 41470m + 37183)$
22		$(2, 29, 90090m + 21747)$
23	$(19, 55, 1)$	$(4, 87, 15015m + 8802)$
24	$(2, 3, 5)$	$(2, 29, 90090m + 83877)$
25	$(43, 85, 1)$	$(5, 29, 36036m + 9941)$
26	$(3, 13, 3)$	$(3, 13, 133980m + 83163)$
27	$(4, 5, 16)$	$(3, 29, 60060m + 57989)$
28		$(2, 29, 90090m + 27959)$

For $p = 180180(31m + i) + 1$ $(i = 1, \ldots, 30)$:

p mod 31	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2	$(11, 171, 3)$	
3	$(61, 61, 1)$	$(31, 91, 1980m + 511)$
4		
5	$(4, 7, 10)$	$(3, 93, 20020m + 10333)$
6		
7		
8		
9	$(4, 4, 2)$	
10	$(4, 16, 16)$	
11	$(15, 29, 1)$	$(21, 31, 8580m + 2491)$
12	$(2, 4, 4)$	$(6, 186, 5005m + 2099)$
13	$(16, 16, 4)$	
14	$(3, 7, 3)$	$(3, 7, 265980m + 180183)$
15	$(7, 7, 19)$	
16	$(7, 10, 4)$	$(31, 39, 4620m + 4322)$
17	$(6, 16, 1)$	$(3, 31, 60060m + 3875)$
18	$(5, 25, 1)$	$(5, 93, 12012m + 2325)$
19		$(13, 31, 13860m + 4471)$
20		$(4, 93, 15015m + 6781)$
21	$(3, 21, 1)$	$(11, 31, 16380m + 9511)$
22	$(4, 10, 7)$	$(10, 31, 18018m + 12787)$
23	$(3, 3, 7)$	$(3, 3, 620620m + 520527)$
24	$(2, 16, 1)$	$(4, 31, 45045m + 43592)$
25	$(7, 9, 1)$	$(7, 31, 25740m + 2491)$
26		$(6, 31, 30030m + 6781)$
27	$(4, 8, 1)$	$(5, 31, 36036m + 12787)$
28		$(4, 31, 45045m + 21796)$
29	$(2, 8, 2)$	$(3, 31, 60060m + 36811)$
30	$(5, 5, 5)$	$(2, 31, 90090m + 66841)$

For $p = 180180(37m + i) + 1$ ($i = 1, \ldots, 36$):

$p \mod 37$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
0	$(7, 34, 7)$	
2	$(6, 31, 1)$	
3	$(36, 36, 1)$	
4	$(13, 205, 1)$	
5	$(7, 7, 34)$	
6	$(4, 7, 4)$	$(4, 7, 238095m + 115834)$
7		
8	$(19, 19, 4)$	
9	$(7, 16, 1)$	$(2, 37, 90090m + 34089)$
10	$(2, 19, 1)$	
11		$(3, 37, 60060m + 58437)$
12	$(22, 106, 1)$	
13		
14		
15		
16	$(12, 34, 1)$	
17	$(5, 5, 3)$	$(7, 37, 25740m + 19479)$
18	$(4, 4, 7)$	
19		
20		
21		$(7, 37, 25740m + 24349)$
22		$(4, 37, 45045m + 10957)$
23	$(4, 28, 1)$	$(3, 185, 12012m + 6493)$
24	$(3, 5, 5)$	$(2, 37, 90090m + 75481)$
25	$(3, 25, 1)$	$(13, 37, 13860m + 1873)$
26		
27		
28		$(10, 37, 18018m + 487)$
29	$(5, 15, 1)$	
30		$(5, 37, 36036m + 22401)$
31		$(7, 37, 1980m + 1837)$
32		$(2, 37, 6930m + 5619)$
33		$(5, 37, 2772m + 1873)$
34		$(4, 37, 3465m + 1873)$
35		$(3, 37, 4620m + 1873)$
36		$(2, 37, 6930m + 1837)$

For $p = 13860(41m + i) + 1$ $(i = 1, \ldots, 40)$:

$p \mod 41$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3	$(40, 40, 1)$	
4		
5	$(2, 3, 7)$	$(2, 3, 1231230m + 1141147)$
6		
7		
8		
9	$(2, 7, 3)$	$(2, 7, 527670m + 450453)$
10		
11	$(2, 11, 1)$	$(2, 21, 175890m + 55771)$
12	$(3, 11, 5)$	$(3, 11, 223860m + 10925)$
13	$(3, 4, 24)$	$(3, 4, 615615m + 480504)$
14	$(3, 14, 1)$	$(3, 14, 175890m + 90091)$
15		$(3, 41, 60060m + 14649)$
16	$(6, 7, 1)$	$(6, 7, 175890m + 171601)$
17		$(11, 41, 175890m + 171601)$
18		
19		
20	$(3, 24, 4)$	
21	$(8, 36, 1)$	$(21, 41, 8580m + 5441)$
22		
23	$(3, 32, 3)$	
24	$(5, 33, 1)$	$(5, 33, 44772m + 37129)$
25		
26		
27	$(3, 5, 11)$	$(3, 5, 492492m + 12023)$
28	$(4, 22, 7)$	
29		
30	$(11, 15, 1)$	$(4, 7, 263835m + 57937)$
31	$(3, 3, 32)$	$(11, 41, 16380m + 15581)$
32		
33		
34		
35	$(4, 24, 3)$	
36	$(7, 22, 4)$	$(6, 41, 30030m + 18311)$
37		$(5, 11, 134316m + 9831)$
38	$(5, 11, 3)$	$(5, 11, 134316m + 9831)$
39	$(3, 7, 2)$	$(3, 41, 60060m + 48341)$
40	$(10, 37, 1)$	$(2, 41, 90090m + 48341)$

For $p = 13860(43m + i) + 1$ $(i = 1, \ldots, 42)$:

$p \mod 43$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
0	$(2, 2, 11)$	
2		
3	$(2, 4, 27)$	
4		
5	$(3, 67, 3)$	$(6, 6, 215215m + 45051)$
6		
7		
8	$(5, 69, 1)$	
9		
10		
11		
12	$(10, 13, 1)$	$(10, 13, 59598m + 19405)$
13		
14	$(2, 22, 22)$	
15		
16	$(2, 9, 12)$	$(4, 11, 176085m + 94186)$
17	$(25, 31, 1)$	
18	$(14, 40, 1)$	
19		
20		
21		
22	$(7, 79, 7)$	$(22, 43, 8190m + 2857)$
23	$(7, 37, 1)$	$(7, 129, 8580m + 5587)$
24	$(3, 3, 67)$	$(3, 3, 860860m + 820887)$
25		
26		
27		
28		
29	$(3, 29, 1)$	$(15, 43, 12012m + 5587)$
30		
31		$(13, 43, 13860m + 967)$
32		$(6, 43, 30030m + 11174)$
33	$(2, 22, 1)$	$(2, 22, 176085m + 118756)$
34	$(4, 27, 2)$	$(10, 43, 18018m + 17599)$
35	$(22, 22, 4)$	
36		
37		$(7, 43, 25740m + 22747)$
38	$(2, 27, 4)$	$(2, 129, 30030m + 5587)$
39		$(4, 43, 45045m + 35617)$
40	$(2, 11, 2)$	$(4, 43, 45045m + 35617)$
41	$(16, 121, 1)$	$(3, 43, 60060m + 5587)$
42	$(2, 12, 9)$	$(2, 43, 90090m + 35617)$

For $p = 180180(47m + i) + 1$ $(i = 1, \ldots, 46)$:

$p \mod 47$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3	$(46, 46, 1)$	$(5, 423, 4004m + 2215)$
4	$(6, 17, 6)$	
5		
6		
7	$(15, 22, 1)$	$(15, 22, 25662m + 16927)$
8		
9		
10	$(6, 6, 17)$	$(6, 6, 235235m + 115132)$
11	$(3, 10, 11)$	$(3, 10, 282282m + 216227)$
12		
13		
14		
15	$(23, 45, 1)$	
16	$(3, 16, 1)$	
17	$(3, 3, 21)$	$(3, 3, 940940m + 400421)$
18	$(2, 6, 4)$	$(2, 6, 705705m + 495499)$
19		
20		
21		$(3, 141, 20020m + 10649)$
22	$(2, 8, 3)$	
23		
24		
25		
26	$(2, 3, 8)$	$(2, 3, 1411410m + 1291298)$
27		$(7, 47, 25740m + 4929)$
28	$(2, 12, 2)$	
29	$(3, 8, 2)$	$(3, 11, 256620m + 191110)$
30		
31	$(10, 11, 3)$	$(10, 11, 76986m + 22935)$
32	$(7, 27, 1)$	
33		$(3, 235, 12012m + 10223)$
34	$(9, 21, 1)$	
35	$(31, 91, 1)$	
36	$(2, 24, 1)$	
37	$(3, 4, 4)$	$(3, 4, 705705m + 675679)$
38		
39		
40	$(2, 4, 6)$	
41	$(4, 12, 1)$	
42	$(6, 8, 1)$	$(2, 47, 90090m + 30669)$
43		$(5, 47, 36036m + 22235)$
44	$(7, 9, 3)$	$(7, 9, 134420m + 120123)$
45	$(4, 6, 2)$	$(3, 47, 60060m + 10223)$
46	$(4, 4, 3)$	$(2, 47, 90090m + 40253)$

For $p = 180180(53m + i) + 1$ $(i = 1, \ldots, 52)$:

$p \mod 53$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2	$(3, 24, 67)$	
3	$(4, 16, 82)$	
4		
5		
6	$(7, 91, 1)$	
7		
8		
9	$(3, 11, 45)$	$(5, 159, 12012m + 9519)$
10	$(5, 85, 1)$	
11		
12	$(3, 6, 3)$	$(3, 6, 530530m + 180183)$
13	$(2, 7, 19)$	$(2, 7, 682110m + 128719)$
14	$(2, 27, 1)$	
15	$(3, 9, 2)$	
16		$(6, 318, 5005m + 3683)$
17	$(4, 82, 16)$	
18	$(3, 18, 1)$	
19		
20		
21	$(6, 9, 1)$	
22	$(7, 7, 13)$	
23		
24	$(13, 49, 1)$	
25		
26		
27		
28	$(7, 19, 2)$	
29		
30	$(3, 3, 6)$	$(3, 3, 1061060m + 660666)$
31	$(2, 19, 7)$	
32	$(4, 31, 3)$	
33	$(4, 40, 1)$	$(7, 159, 8580m + 1457)$
34	$(17, 25, 1)$	
35	$(5, 5, 17)$	
36		
37	$(2, 3, 9)$	$(2, 3, 1591590m + 900909)$
38	$(2, 9, 3)$	$(2, 9, 530530m + 220223)$
39		$(3, 53, 60060m + 15865)$
40	$(11, 45, 3)$	$(14, 53, 12870m + 1457)$
41		$(2, 159, 30030m + 28897)$
42	$(10, 16, 1)$	$(4, 159, 15015m + 12182)$
43	$(4, 4, 10)$	
44	$(3, 67, 24)$	
45		

$p \mod 53$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
46		
47	$(3, 31, 4)$	
48		$(2, 159, 30030m + 27197)$
49		$(5, 53, 36036m + 27197)$
50	$(5, 17, 5)$	
51	$(4, 10, 4)$	$(3, 53, 60060m + 27197)$
52		$(2, 53, 90090m + 27197)$

For $p = 180180(59m + i) + 1$ $(i = 1, \ldots, 58)$:

$p \mod 59$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3		
4		
5	$(4, 19, 7)$	
6	$(25, 85, 1)$	
7		
8	$(2, 5, 6)$	$(2, 5, 1063062m + 864870)$
9		
10	$(3, 3, 46)$	$(3, 3, 1181180m + 560606)$
11	$(3, 5, 4)$	$(3, 5, 708708m + 216220)$
12	$(3, 93, 11)$	
13		
14		
15		$(5, 59, 36036m + 22599)$
16	$(2, 3, 10)$	$(2, 3, 1771770m + 810820)$
17		
18	$(29, 57, 1)$	
19	$(2, 6, 5)$	$(2, 6, 885885m + 840845)$
20	$(3, 20, 1)$	$(3, 20, 177177m + 138139)$
21		
22	$(4, 15, 1)$	$(4, 15, 177177m + 78079)$
23	$(5, 12, 1)$	$(5, 12, 177177m + 48049)$
24	$(4, 31, 10)$	
25	$(5, 26, 5)$	$(3, 11, 322140m + 300393)$
26	$(5, 5, 26)$	
27	$(4, 37, 2)$	
28	$(10, 31, 4)$	
29	$(3, 4, 5)$	$(3, 4, 885885m + 225230)$
30		
31	$(2, 4, 31)$	
32	$(4, 6, 32)$	
33	$(3, 46, 3)$	
34	$(11, 93, 3)$	
35	$(2, 15, 2)$	$(2, 15, 354354m + 84086)$

$p \mod 59$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
36		
37	$(5,6,2)$	$(5,6,354354m+318320)$
38	$(4,7,19)$	$(4,7,379665m+276724)$
39		$(7,59,25740m+14397)$
40		
41		
42		
43	$(4,5,3)$	$(4,5,531531m+468471)$
44		
45	$(2,30,1)$	$(3,59,60060m+32575)$
46	$(4,32,6)$	
47	$(16,24,2)$	
48		
49	$(7,17,1)$	
50		
51		
52	$(2,37,4)$	$(4,59,45045m+16033)$
53	$(6,10,1)$	$(6,10,177177m+33034)$
54	$(22,169,1)$	$(2,59,90090m+1527)$
55	$(4,10,31)$	$(5,59,36036m+30539)$
56	$(3,10,2)$	$(3,10,354354m+240242)$
57	$(2,10,3)$	$(3,59,60060m+30539)$
58		$(2,59,90090m+30539)$

For $p = 180180(61m+i)+1$ $(i=1,\ldots,60)$:

$p \mod 61$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
2	$(11,111,1)$	
3	$(2,9,17)$	$(2,9,610610m+260277)$
4	$(6,51,1)$	
5		
6		
7	$(3,95,3)$	
8		
9	$(22,25,1)$	
10		
11		
12		
13	$(9,17,2)$	
14		
15		

$p \mod 61$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
16	$(2, 31, 1)$	
17		$(9, 61, 20020m + 8205)$
18		
19		
20	$(4, 31, 31)$	
21		
22		
23		$(10, 61, 18018m + 12406)$
24		
25	$(31, 31, 4)$	
26		
27	$(15, 57, 1)$	
28		
29	$(2, 17, 9)$	
30		
31		
32		$(61, 91, 1980m + 1201)$
33	$(13, 169, 1)$	
34		
35		
36	$(5, 49, 1)$	
37		
38	$(4, 46, 1)$	
39	$(7, 7, 5)$	
40		
41	$(3, 41, 1)$	$(21, 61, 8580m + 4501)$
42	$(7, 35, 1)$	
43		
44		
45		
46		
47	$(3, 3, 95)$	$(3, 305, 12012m + 9649)$
48		$(2, 427, 12870m + 211)$
49		$(13, 61, 13860m + 3181)$
50		
51		$(11, 61, 16380m + 10741)$
52		$(10, 61, 18018m + 15655)$
53		
54		
55		$(7, 61, 25740m + 13081)$
56		$(6, 61, 30030m + 21661)$
57		$(5, 61, 36036m + 33673)$
58		$(4, 61, 45045 + 6646)$
59		$(3, 61, 60060m + 21661)$
60	$(5, 7, 7)$	$(2, 61, 90090m + 51691)$

For $p = 180180(67m + i) + 1$ $(i = 1, \ldots, 66)$:

$p \mod 67$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3	$(66, 66, 1)$	
4		
5		
6		
7		
8	$(7, 115, 1)$	
9		
10		
11		
12		
13	$(3, 5, 9)$	$(3, 5, 804804m + 576585)$
14	$(2, 24, 7)$	
15	$(12, 14, 2)$	
16		
17	$(16, 21, 1)$	
18	$(4, 196, 1)$	
19		$(3, 67, 60060m + 4483)$
20	$(5, 5, 59)$	
21	$(3, 9, 5)$	
22		
23	$(6, 6, 121)$	$(6, 6, 335335m + 105226)$
24		
25	$(4, 17, 1)$	$(3, 3, 1341340m + 580595)$
26	$(22, 64, 1)$	
27	$(25, 193, 1)$	
28	$(2, 4, 42)$	
29		
30	$(3, 15, 3)$	$(2, 14, 431145m + 315327)$
31	$(5, 9, 3)$	$(5, 9, 268268m + 212215)$
32	$(4, 12, 7)$	
33	$(11, 61, 1)$	
34		
35		
36		
37		
38	$(6, 56, 1)$	
39		
40	$(2, 7, 24)$	$(2, 7, 862290m + 283164)$
41		$(3, 67, 60060m + 23307)$
42		
43		
44	$(2, 12, 14)$	$(7, 67, 25740m + 14599)$
45	$(3, 45, 1)$	

$p \mod 67$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
46		
47	$(13,31,1)$	$(7,67,25740m+19209)$
48		
49		
50		
51	$(2,34,1)$	$(4,7,431145m+424722)$
52		$(4,67,45045m+2017)$
53	$(5,27,1)$	$(3,67,60060m+6275)$
54		
55		$(13,67,13860m+3103)$
56		$(12,67,15015m+4258)$
57	$(9,15,1)$	
58	$(7,24,2)$	$(10,67,18018m+7261)$
59	$(2,42,4)$	$(2,42,143715m+66499)$
60		
61		$(7,67,25740m+14983)$
62	$(2,17,2)$	$(2,67,90090m+57819)$
63	$(19,127,1)$	$(5,67,36036m+25279)$
64	$(7,12,4)$	$(4,67,45045m+34288)$
65		$(3,67,60060m+49303)$
66		$(2,67,90090m+79333)$

For $p = 180180(71m+i)+1$ $(i = 1,\ldots,70)$:

$p \mod 71$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
2		
3	$(141,141,1)$	
4		
5	$(4,16,10)$	
6		$(6,71,30030m+21571)$
7	$(3,11,28)$	$(3,11,387660m+256648)$
8		
9	$(23,34,1)$	$(11,28,41535m+22818)$
10		
11		
12		
13	$(2,11,42)$	$(2,11,581490m+188412)$
14	$(2,4,9)$	$(3,5,852852m+228247)$
15		
16	$(3,8,3)$	
17		
18		
19		
20	$(3,12,2)$	$(3,12,355355m+330332)$

$p \mod 71$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
21	$(6,6,2)$	$(6,6,355355m+310312)$
22		
23		
24	$(3,24,1)$	
25		
26		
27	$(2,9,4)$	$(5,71,36036m+19287)$
28	$(3,17,39)$	
29	$(8,9,1)$	
30	$(3,4,6)$	$(3,4,1066065m+390396)$
31	$(11,42,2)$	$(11,42,27690m+8582)$
32		
33	$(2,12,3)$	$(2,154,41535m+8193)$
34	$(4,6,3)$	
35		
36	$(10,64,1)$	$(36,71,5005m+141)$
37	$(4,10,16)$	$(6,21,101530m+98701)$
38	$(4,4,40)$	
39	$(7,61,1)$	
40	$(2,6,6)$	$(2,6,1066065m+855861)$
41		
42	$(2,18,2)$	$(2,18,355355m+245247)$
43		
44	$(2,42,11)$	$(2,42,152295m+87956)$
45	$(16,40,1)$	
46		
47	$(7,7,29)$	
48		
49		
50	$(5,125,5)$	
51	$(3,28,11)$	$(21,71,8580m+1571)$
52		
53		
54	$(2,36,1)$	
55	$(3,3,8)$	$(3,3,1421420m+1361368)$
56	$(15,19,1)$	
57		$(15,71,12012m+10151)$
58	$(17,39,3)$	$(2,497,12870m+10151)$
59		
60	$(2,3,12)$	$(4,71,45045m+30453)$

$p \mod 71$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
61	$(11,13,1)$	$(3,39,109340m+67777)$
62	$(4,18,1)$	
63	$(4,40,4)$	
64	$(6,12,1)$	
65		
66	$(5,19,3)$	$(6,71,30030m+10151)$
67	$(7,29,7)$	$(5,71,36036m+10151)$
68	$(4,9,2)$	$(4,9,355355m+80082)$
69		$(3,71,60060m+10151)$
70	$(3,6,4)$	$(2,71,90090m+10151)$

For $p = 180180(73m+i)+1$ $(i = 1, \ldots, 72)$:

$p \mod 73$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
2		
3		
4		
5	$(6,61,1)$	
6		
7		
8		$(2,219,30030m+2057)$
9	$(4,55,1)$	$(4,55,59787m+30304)$
10		$(4,73,45045m+42577)$
11		
12		
13		
14	$(5,117,1)$	
15		
16		
17		
18		
19	$(2,37,1)$	
20		
21		
22		
23	$(8,8,8)$	
24		
25		
26		
27	$(11,31,3)$	
28		
29		
30	$(5,257,5)$	

$p \mod 73$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
31	$(9,21,17)$	
32		
33		
34	$(7,7,3)$	
35		
36	$(4,115,10)$	
37		
38	$(9,65,1)$	
39	$(8,64,1)$	
40		
41	$(5,9,13)$	
42		
43	$(31,106,1)$	
44		
45		
46	$(13,45,1)$	
47		$(3,219,20020m+3291)$
48		
49	$(3,49,1)$	
50		
51		
52		
53		
54	$(3,7,7)$	$(3,7,626340m+145867)$
55	$(17,21,9)$	
56	$(5,5,257)$	$(7,949,1980m+217)$
57	$(5,13,9)$	
58	$(36,71,1)$	
59	$(10,22,1)$	$(3,73,60060m+25505)$
60		
61	$(7,21,1)$	$(13,73,13860m+4177)$
62		
63	$(29,141,1)$	
64		$(10,73,18018m+11107)$
65		
66		$(4,73,45045m+22214)$
67		$(7,73,25740m+23977)$
68	$(10,10,46)$	$(2,219,30030m+11107)$
69		$(5,73,36036m+29125)$
70	$(9,13,5)$	$(4,73,45045m+11107)$
71		$(3,73,60060m+41137)$
72	$(12,67,1)$	$(2,73,90090m+11107)$

For $p = 180180(79m + i) + 1$ ($i = 1, \ldots, 78$):

$p \mod 79$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3	$(78, 78, 1)$	
4		
5		
6		
7	$(2, 4, 10)$	
8		
9	$(3, 123, 3)$	
10	$(2, 9, 22)$	$(2, 9, 790790m + 120142)$
11		
12		
13		
14	$(3, 3, 123)$	$(3, 3, 1581580m + 1401523)$
15		
16		
17		
18	$(11, 36, 1)$	$(11, 36, 35945m + 22296)$
19		
20		
21	$(7, 34, 1)$	
22		
23	$(4, 10, 2)$	
24		
25		
26	$(2, 5, 8)$	$(2, 5, 1423422m + 126134)$
27		
28		
29		
30	$(2, 10, 4)$	$(2, 10, 711711m + 585589)$
31	$(5, 16, 1)$	
32	$(6, 11, 6)$	$(6, 11, 215670m + 40956)$
33		
34		
35	$(22, 202, 4)$	$(9, 395, 4004m + 963)$
36		
37		
38	$(13, 73, 1)$	
39		

$p \mod 79$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
40	$(31, 51, 1)$	
41		
42		
43		
44		
45	$(6, 66, 1)$	$(6, 66, 35945m + 2731)$
46		
47		
48		
49		
50		
51	$(2, 24, 107)$	
52	$(4, 4, 5)$	$(6, 6, 395395m + 340351)$
53	$(3, 53, 1)$	
54		
55		
56		
57		
58		
59	$(19, 25, 1)$	$(7, 79, 25740m + 16617)$
60	$(2, 40, 1)$	
61		
62		
63	$(24, 107, 2)$	
64		
65	$(43, 463, 1)$	$(3, 395, 12012m + 8971)$
66		$(14, 79, 12870m + 5539)$
67		$(13, 79, 13860m + 1579)$
68		$(7, 79, 25740m + 20527)$
69	$(4, 20, 1)$	
70	$(9, 22, 2)$	$(10, 79, 18018m + 2965)$
71		
72	$(8, 10, 1)$	
73	$(2, 20, 2)$	$(7, 79, 25740m + 5539)$
74		$(2, 79, 90090m + 80967)$
75		$(5, 79, 36036m + 20983)$
76	$(5, 8, 2)$	$(4, 79, 45045m + 11974)$
77	$(2, 22, 9)$	$(3, 79, 60060m + 57019)$
78	$(2, 8, 5)$	$(2, 79, 90090m + 57019)$

For $p = 180180(83m + i) + 1$ $(i = 1, \ldots, 82)$:

$p \mod 83$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3		
4	$(11, 151, 1)$	
5		
6		
7		
8		
9		
10	$(3, 4, 7)$	$(3, 4, 1246245m + 660667)$
11	$(2, 3, 14)$	$(2, 3, 2492490m + 360374)$
12	$(15, 24, 3)$	
13		
14		
15		
16	$(5, 133, 1)$	
17		
18	$(3, 31, 25)$	
19	$(4, 7, 3)$	$(4, 7, 534105m + 32178)$
20	$(3, 25, 31)$	
21	$(13, 115, 1)$	
22		
23	$(2, 7, 6)$	$(2, 7, 1068210m + 553416)$
24	$(41, 81, 1)$	
25		
26	$(10, 25, 1)$	$(3, 15, 332332m + 120144)$
27		
28	$(3, 28, 1)$	$(3, 28, 178035m + 105106)$
29		
30	$(49, 61, 1)$	
31	$(4, 21, 1)$	$(4, 21, 178035m + 77221)$
32	$(3, 7, 4)$	$(3, 7, 712140m + 34324)$
33		
34	$(7, 12, 1)$	$(7, 12, 178035m + 49336)$
35		
36		
37		
38	$(4, 52, 2)$	
39		$(9, 83, 20020m + 6995)$
40	$(7, 7, 61)$	

$p \mod 83$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
41	$(2,6,7)$	$(2,6,1246245m+720727)$
42		
43		
44		
45		
46		
47		
48	$(3,3,37)$	$(3,3,1661660m+1461497)$
49	$(2,21,2)$	$(2,21,356070m+175892)$
50		
51	$(3,14,2)$	$(3,14,356070m+257402)$
52		
53		
54	$(4,6,45)$	
55		
56		
57	$(22,34,1)$	$(3,249,20020m+8201)$
58		
59		
60		
61		
62	$(9,37,1)$	
63	$(2,42,1)$	$(2,42,178035m+17161)$
64	$(16,26,1)$	$(6,6,415415m+295325)$
65		
66	$(3,24,15)$	
67		
68		
69		
70	$(25,31,3)$	
71		
72	$(2,4,52)$	
73	$(2,52,4)$	
74	$(3,37,3)$	
75	$(6,14,1)$	$(6,14,178035m+83656)$
76		
77		
78		$(2,249,30030m+9407)$
79		$(5,83,36036m+33431)$
80	$(2,14,3)$	$(2,14,534105m+289578)$
81		$(3,83,60060m+9407)$
82		$(2,83,90090m+69467)$

For $p = 180180(89m + i) + 1$ $(i = 1, \dots, 88)$:

$p \mod 89$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3		
4		
5		
6		
7		
8	$(7, 7, 109)$	
9		
10		
11	$(4, 67, 1)$	
12		
13		
14	$(3, 6, 5)$	$(3, 6, 890890m + 630635)$
15		
16	$(3, 3, 10)$	$(3, 3, 1781780m + 1181190)$
17		
18		
19	$(2, 15, 3)$	$(2, 15, 534534m + 318321)$
20	$(3, 10, 3)$	$(3, 10, 534534m + 306309)$
21		
22		
23	$(2, 45, 1)$	$(2, 45, 178178m + 90091)$
24		$(2, 89, 90090m + 43527)$
25	$(3, 15, 2)$	$(3, 15, 356356m + 164166)$
26		
27		
28		
29	$(9, 17, 32)$	
30	$(3, 30, 1)$	$(3, 30, 178178m + 62063)$
31	$(12, 34, 12)$	
32		
33		
34		
35	$(5, 18, 1)$	$(5, 18, 178178m + 42043)$
36	$(6, 15, 1)$	$(6, 15, 178178m + 38039)$
37	$(9, 10, 1)$	$(9, 10, 178178m + 34035)$
38		
39		
40	$(17, 32, 9)$	
41		
42		
43		
44		
45	$(11, 81, 1)$	$(45, 89, 4004m + 45)$

$p \mod 89$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
46		
47		
48	$(8,78,1)$	
49		
50	$(2,3,15)$	$(2,3,2672670m+2402415)$
51		
52	$(3,4,52)$	$(3,4,1336335m+1141192)$
53		
54	$(45,45,4)$	
55	$(29,43,1)$	
56	$(5,9,2)$	$(5,9,356356m+272274)$
57		
58		
59	$(3,7,17)$	$(3,7,763620m+531977)$
60		
61	$(3,52,4)$	$(3,52,102795m+66994)$
62	$(2,5,9)$	$(2,5,1603602m+1009017)$
63	$(3,5,6)$	$(3,89,60060m+36441)$
64		
65	$(5,6,3)$	$(5,6,534534m+300303)$
66	$(4,49,5)$	
67		
68		
69		$(7,89,25740m+12147)$
70	$(133,265,1)$	
71		
72	$(11,21,1)$	
73	$(2,9,5)$	$(2,9,890890m+340345)$
74		
75	$(7,109,7)$	$(3,445,12012m+4049)$
76	$(4,52,3)$	$(2,89,90090m+28343)$
77		
78		$(12,89,15015m+4049)$
79	$(5,49,4)$	$(4,445,9009m+2227)$
80	$(4,5,49)$	$(4,5,801801m+180229)$
81		
82	$(22,85,1)$	
83		
84	$(7,17,3)$	$(2,89,90090m+12147)$
85		$(5,89,36036m+4049)$
86		
87	$(3,17,7)$	$(3,89,60060m+4049)$
88		$(2,89,90090m+4049)$

For $p = 180180(97m + i) + 1$ $(i = 1, \ldots, 96)$:

p mod 97	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3		
4	$(8, 85, 1)$	
5		
6		
7	$(6, 81, 1)$	$(7, 97, 25740m + 21229)$
8		
9		
10	$(11, 353, 3)$	
11	$(3, 151, 3)$	
12	$(4, 73, 1)$	
13		
14		
15		
16		
17		
18	$(19, 46, 1)$	
19		
20	$(4, 7, 52)$	$(4, 7, 624195m + 173797)$
21	$(4, 10, 17)$	
22		
23		
24	$(2, 9, 27)$	$(2, 9, 970970m + 480507)$
25	$(2, 49, 1)$	
26		
27		
28		
29	$(7, 7, 2)$	
30		
31		
32	$(4, 49, 49)$	
33		
34		
35		
36		
37	$(3, 3, 151)$	$(3, 3, 1941940m + 1841991)$
38		
39		
40	$(7, 14, 1)$	

$p \mod 97$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
41		
42	$(4,52,7)$	
43	$(27,247,4)$	
44		
45	$(5,13,3)$	$(5,13,268884m+102567)$
46	$(2,27,9)$	
47		
48		
49		
50		
51		$(6,97,30030m+6192)$
52		
53		$(5,291,12012m+9783)$
54	$(9,27,2)$	
55		
56		
57		
58		$(4,97,45045m+37615)$
59		
60	$(2,7,7)$	$(2,7,1248390m+553417)$
61		
62		
63		
64		
65	$(3,65,1)$	$(3,65,89628m+41581)$
66		
67		
68		
69	$(3,5,13)$	$(3,5,1165164m+792805)$
70	$(21,37,1)$	
71		
72		
73	$(10,17,4)$	
74		
75		
76		
77	$(5,39,1)$	$(7,97,25740m+2919)$
78		
79		
80		

$p \mod 97$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
81		
82		$(4, 97, 45045m + 6037)$
83		
84		$(2, 679, 12870m + 9553)$
85	$(13, 15, 1)$	$(13, 97, 13860m + 7573)$
86	$(7, 52, 4)$	$(4, 291, 15015m + 5263)$
87		
88		$(10, 97, 18018m + 17275)$
89		
90		
91		$(7, 97, 25740m + 9553)$
92		$(2, 97, 90090m + 15789)$
93	$(3, 11, 535)$	$(5, 97, 36036m + 35293)$
94	$(16, 91, 1)$	$(4, 97, 45045m + 35293)$
95	$(3, 13, 5)$	$(3, 97, 60060m + 35293)$
96		$(2, 97, 90090m + 35293)$

For $p = 180180(101m + i) + 1$ $(i = 1, \ldots, 100)$:

$p \mod 101$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
2	$(10, 91, 1)$	$(10, 91, 19998m + 4951)$
3	$(100, 100, 1)$	
4		
5	$(3, 4, 59)$	$(3, 4, 1516515m + 1501559)$
6		
7		
8		
9		
10		
11	$(13, 70, 1)$	$(13, 70, 19998m + 9505)$
12		
13		
14		
15		
16		
17		
18		
19		
20	$(4, 4, 19)$	

$p \mod 101$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
21		
22		
23		
24	$(11,46,1)$	
25		
26	$(2,51,1)$	
27		
28	$(2,3,17)$	$(2,3,3033030m+2072087)$
29		
30		
31		
32		
33		
34	$(3,34,1)$	
35	$(16,19,1)$	
36		$(6,1111,2730m+1811)$
37		
38		
39	$(3,15,9)$	$(3,15,404404m+164173)$
40		
41	$(6,17,1)$	
42	$(25,97,1)$	
43		
44		
45		
46		
47		
48		
49	$(3,59,4)$	
50	$(3,9,15)$	
51		
52		
53		
54		
55		
56		
57		$(9,505,4004m+3449)$
58	$(43,148,1)$	
59		
60	$(5,81,1)$	

$p \mod 101$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
61	$(4,59,3)$	
62	$(3,17,2)$	
63	$(4,76,1)$	
64	$(7,10,13)$	$(7,10,259974m+154453)$
65	$(9,9,5)$	
66		
67		
68		
69		
70		
71	$(67,199,1)$	
72	$(2,17,3)$	$(3,5,1213212m+696723)$
73		
74	$(4,17,52)$	
75		
76	$(9,45,1)$	
77	$(4,19,4)$	
78		
79	$(3,3,45)$	$(3,3,2022020m+620665)$
80		
81		$(21,101,8580m+6881)$
82	$(15,27,1)$	
83		
84		
85	$(7,29,1)$	$(7,130,19998m+15841)$
86	$(10,13,7)$	$(10,13,139986m+5551)$
87		$(3,101,60060m+17245)$
88	$(17,52,4)$	
89		
90	$(3,45,3)$	
91		$(11,101,16380m+4541)$
92		
93		
94	$(5,27,3)$	$(4,101,45045m+892)$
95		
96	$(9,15,3)$	$(6,101,30030m+15461)$
97		$(5,101,36036m+27473)$
98		
99		$(3,101,60060m+45491)$
100	$(5,9,9)$	$(2,101,90090m+45491)$

For $p = 180180(103m + i) + 1$ ($i = 1, \ldots, 102$):

p mod 103	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3		
4		
5		
6		
7		
8	$(6, 21, 9)$	$(6, 21, 147290m + 102969)$
9	$(4, 58, 4)$	
10		
11	$(9, 21, 6)$	
12		
13		$(7, 1339, 1980m + 1807)$
14		
15		
16		
17		
18	$(5, 5, 33)$	
19		
20	$(5, 165, 1)$	
21		
22		
23		
24		
25		
26		
27		
28		
29		
30	$(4, 13, 2)$	$(4, 13, 356895m + 13862)$
31		
32	$(6, 186, 6)$	
33	$(10, 31, 1)$	
34	$(4, 52, 52)$	
35		
36		
37		
38	$(34, 100, 1)$	
39	$(2, 13, 4)$	$(2, 13, 713790m + 159394)$
40		

$p \mod 103$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
41		
42		
43	$(8,13,1)$	
44	$(2,4,13)$	
45		
46	$(3,23,3)$	
47		
48		
49		
50		
51	$(6,9,21)$	
52		
53		
54		
55	$(37,49,5)$	
56		
57		
58		
59	$(6,86,1)$	$(5,103,36036m+2799)$
60	$(3,3,23)$	$(3,3,2062060m+660683)$
61		
62		
63	$(5,33,5)$	
64		
65		
66		
67		
68	$(16,58,1)$	$(4,927,5005m+1312)$
69	$(3,69,1)$	
70		
71		
72	$(7,59,1)$	
73		
74		
75	$(4,4,58)$	
76	$(25,33,1)$	
77		
78	$(2,52,1)$	
79		
80	$(5,49,37)$	

$p \mod 103$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
81		
82		
83	$(52,52,4)$	$(7,309,8580m+7747)$
84		
85		
86		
87		
88	$(61,76,1)$	
89	$(9,23,1)$	$(3,515,12012m+4315)$
90	$(4,26,1)$	
91		$(13,103,13860m+11707)$
92		
93		
94	$(5,37,49)$	$(10,103,18018m+1032$
95	$(2,26,95)$	$(2,26,356895m+291062)$
96		
97		$(7,103,25740m+7747)$
98	$(6,6,186)$	$(6,6,515515m+280466)$
99		$(5,103,36036m+28339)$
100		$(4,103,45045m+1312)$
101	$(22,295,1)$	$(3,103,60060m+16327)$
102		$(2,103,90090m+46357)$

For $p = 180180(107m+i)+1$ $(i = 1, \dots, 106)$:

$p \mod 107$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
2		
3	$(106,106,1)$	
4	$(16,301,1)$	
5		
6		
7	$(4,58,6)$	$(4,7,688545m+424882)$
8	$(4,7,172)$	
9		
10		
11		
12	$(3,10,25)$	$(3,10,642642m+72097)$
13		
14		
15		
16		
17		
18	$(5,5,33)$	
19	$(10,25,3)$	
20		

$p \mod 107$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
21	$(2,6,9)$	$(2,6,1606605m+765774)$
22	$(2,4,67)$	
23	$(15,50,1)$	$(3,12,535535m+320323)$
24	$(3,12,3)$	
25		
26		
27	$(3,25,10)$	
28		
29	$(7,46,1)$	
30	$(3,18,2)$	
31		
32	$(6,9,2)$	
33		
34		
35	$(7,9,17)$	$(7,9,306020m+217377)$
36	$(3,36,1)$	$(3,143,44940m+3781)$
37	$(2,97,37)$	
38		
39		
40	$(4,27,1)$	
41	$(3,3,12)$	$(3,3,2142140m+2042052)$
42		$(2,107,90090m+29469)$
43		
44		
45	$(2,3,18)$	$(2,3,3213210m+1441458)$
46	$(2,29,24)$	
47		
48		
49	$(4,67,2)$	
50		
51		
52		
53		
54		
55	$(24,29,2)$	
56		
57		
58		
59		
60		
61		
62		
63	$(2,27,2)$	
64		
65		

$p \mod 107$	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
66		
67	$(4,6,58)$	
68		
69		
70		
71		
72	$(7,172,4)$	
73	$(21,51,1)$	
74		
75	$(71,211,1)$	
76		
77		
78	$(4,9,3)$	$(4,9,535535m+420423)$
79		
80		
81	$(2,54,1)$	
82		
83		
84	$(3,9,4)$	$(11,39,44940m+1261)$
85	$(5,43,1)$	
86	$(13,33,1)$	$(13,33,44940m+34861)$
87		$(7,107,25740m+3849)$
88		
89		
90		
91	$(3,6,6)$	$(3,6,1071070m+690696)$
92	$(2,24,29)$	
93	$(40,313,1)$	$(3,535,12012m+4715)$
94	$(2,67,4)$	$(2,107,90090m+69041)$
95	$(6,58,4)$	
96	$(3,4,9)$	$(3,4,1606605m+825834)$
97	$(6,18,1)$	
98		
99		
100		
101	$(2,37,94)$	
102	$(11,13,3)$	$(2,107,90090m+68199)$
103	$(2,18,3)$	$(5,107,36036m+4715)$
104		
105	$(6,6,3)$	$(3,107,60060m+52763)$
106	$(2,9,6)$	$(2,107,90090m+22733)$

For $p = 180180(109m + i) + 1$ ($i = 1, \ldots, 108$):

$p \mod 109$	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
2		
3		
4		
5		
6		
7		
8	$(6, 91, 1)$	$(6, 91, 35970m + 24751)$
9	$(2, 39, 7)$	$(2, 39, 251790m + 90097)$
10	$(5, 48, 5)$	$(3, 15, 436436m + 12075)$
11	$(5, 5, 48)$	
12		
13		
14	$(7, 187, 1)$	
15	$(6, 10, 20)$	$(6, 10, 327327m + 123143)$
16	$(4, 5, 60)$	$(4, 5, 981981m + 45105)$
17		
18		
19		$(7, 109, 25740m + 1417)$
20	$(25, 157, 1)$	
21		
22		
23		
24		
25		
26		
27	$(2, 5, 11)$	$(2, 5, 1963962m + 810821)$
28	$(2, 55, 1)$	$(2, 55, 178542m + 14743)$
29	$(13, 151, 1)$	
30		
31		
32		
33	$(5, 16, 15)$	$(21, 26, 35970m + 15511)$
34		
35		
36	$(15, 63, 3)$	
37	$(3, 63, 15)$	
38		
39		
40	$(36, 106, 1)$	

p mod 109	(a,b,c) satisfying (3)	(a,b,c) satisfying (2)
41		
42		
43	$(5,22,1)$	$(5,22,178542m+22933)$
44		$(2,1199,8190m+6537)$
45	$(27,105,1)$	
46	$(10,11,1)$	$(10,11,178542m+24571)$
47		
48		
49		
50	$(15,16,5)$	
51		
52		
53	$(2,11,5)$	$(2,11,892710m+737105)$
54	$(43,71,1)$	
55	$(12,100,1)$	
56		
57		
58	$(9,97,1)$	
59		
60		
61		
62		
63		
64		
65		$(9,109,20020m+17265)$
66		
67		
68	$(4,82,1)$	$(2,7,1402830m+1222689)$
69		
70		
71		
72		
73	$(3,73,1)$	
74		
75	$(5,60,4)$	
76	$(5,15,16)$	
77		
78		
79		
80		

p mod 109	(a, b, c) satisfying (3)	(a, b, c) satisfying (2)
81		
82		
83		$(3, 327, 20020m + 18367)$
84		
85		
86		
87		
88		
89		$(7, 109, 25740m + 24087)$
90	$(19, 23, 1)$	
91		
92		
93		
94		
95	$(7, 39, 2)$	$(7, 39, 71940m + 68642)$
96		
97		$(13, 109, 13860m + 4069)$
98		$(6, 109, 30030m + 28928)$
99	$(4, 60, 5)$	
100		$(10, 109, 18018m + 5455)$
101		
102		
103		$(7, 109, 25740m + 8029)$
104		$(2, 109, 90090m + 88437)$
105	$(5, 11, 2)$	$(5, 109, 36036m + 23473)$
106		$(4, 109, 45045m + 14464)$
107	$(6, 20, 10)$	$(3, 109, 60060m + 59509)$
108		$(2, 109, 90090m + 59509)$