NARAYANA NUMBERS WHICH ARE CONCATENATIONS OF TWO BASE b REPDIGITS

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Abstract

In this study, all Narayana numbers which are concatenations of two base b repdigits are found. Lower bounds for linear forms in logarithms of algebraic numbers and a modified Baker-Davenport reduction technique are used in the main result's proof.

1. Introduction

For an integer $b \geq 2$, a positive integer R is called a *base b repdigit* if it has only one distinct digit in its base b representations. In particular, such a number has the form $a\left(\frac{b^m-1}{b-1}\right)$ for some positive integers a, m with $m \geq 1$ and $0 \leq a \leq b-1$. When $b = 10$, we omit the base and simply say R is a repdigit.

Given positive integers $A_1, A_2, \cdots, A_t$, the concatenation of their base b strings of digits is $\overline{A_1A_2\cdots A_t}_{(b)}$. A base b repdigit R is of the form $R = \overline{\underbrace{a\ldots a}_{m \text{ times}}}_{(b)}$, whereas concatenation of two base b repdigits is $\overline{\underbrace{a_1\ldots a_1}_{l \text{ times}}\underbrace{a_2\ldots a_2}_{m \text{ times}}}_{(b)}$, where $a_1, a_2 \in \{0, 1, \ldots, b-1\}$ with $a_1 > 0$.

Diophantine equations with repdigits and terms from linear recurrent sequences like Fibonacci, Lucas, Pell, Pell-Lucas, balancing, and Lucas-balancing sequences have recently gained much attention from researchers. The terms of various binary and ternary recurrent sequences, as well as the sum, difference, product, and concatenations of repdigits are covered in a number of works. For example, Lucas, Pell, and Pell-Lucas numbers as the sum of two repdigits have been studied in [4] and [5]. Bravo et al. [7] obtained all base b repdigits which are the sum of two

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Narayana numbers. Alahmadi et al. [1] found that $13, 21, 34, 55, 59, 144, 233$, and 377 are the only Fibonacci numbers which are concatenations of two repdigits. In [9], Ddamulira studied all the Padovan numbers which are concatenations of two repdigits. For more, one can see [2, 6, 10, 12, 13, 14], and [18]. In our study, we find all Narayana numbers which are concatenations of two base b repdigits, where $2 \le b \le 9$. In particular, we solve the Diophantine equation

$$N_n = \overline{\underbrace{a_1 \dots a_1}_{l \text{ times}} \underbrace{a_2 \dots a_2}_{m \text{ times}}}_{(b)} = a_1 \left(\frac{b^l - 1}{b - 1}\right) \cdot b^m + a_2 \left(\frac{b^m - 1}{b - 1}\right) \tag{1}$$

such that $n, m, l \ge 1$ and $a_1, a_2 \in \{0, 1, \dots, b-1\}$ with $a_1 > 0$ and $a_1 \ne a_2$.

Narayana numbers were derived from a problem involving cows and calves proposed by the Indian mathematician Narayana Pandit[3]. The Narayana's cows sequence $\{N_n\}_{n \ge 0}$ can be expressed as the following ternary linear recurrence sequence:

$$N_{n+3} = N_{n+2} + N_n \tag{2}$$

for $n \ge 0$ with initial condition $(N_0, N_1, N_2) = (0, 1, 1)$. The first few terms of this sequence are

$$0, 1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, \cdots .$$

The characteristic polynomial of the Narayana's cows sequence is $f(x) = x^3 - x^2 - 1$, which is irreducible in $\mathbb{Q}[x]$. The zeros of this polynomial are α (≈ 1.46557) and two conjugate complex zeros β and γ with $|\beta| = |\gamma| < 1$. The following are some properties of the Narayana's sequence (see Lemma 5 in [7]). The Binet formula of it is given by

$$N_n = a\alpha^n + b\beta^n + c\gamma^n \quad \text{for all} \quad n \ge 0,$$

where

$$a = \frac{\alpha}{(\alpha - \beta)(\alpha - \gamma)}, \; b = \frac{\beta}{(\beta - \alpha)(\beta - \gamma)}, \; c = \frac{\gamma}{(\gamma - \alpha)(\gamma - \beta)}.$$

It can be alternatively written as $N_n = C_\alpha \alpha^{n+2} + C_\beta \beta^{n+2} + C_\gamma \gamma^{n+2}$ for all $n \ge 0$, where $C_x = \frac{1}{x^3 + 2}$ for $x \in \{\alpha, \beta, \gamma\}$. The minimal polynomial of C_α is $31x^3 - 31x^2 + 10x - 1$ and all the zeros of this polynomial are inside the unit circle. Numerically, the following estimates hold for α, C_α and $C_\beta \beta^{n+2} + C_\gamma \gamma^{n+2}$:

$$1.45 < \alpha < 1.5; \quad 5 < C_\alpha^{-1} < 5.15; \quad |C_\beta \beta^{n+2} + C_\gamma \gamma^{n+2}| < 1/2 \quad \text{for all} \quad n \ge 1.$$

It is simple to demonstrate by induction that

$$\alpha^{n-2} \le N_n \le \alpha^{n-1} \text{ for all } n \ge 1. \tag{3}$$

Our main result is the following.

Theorem 1. *The only Narayana numbers that are concatenations of two repdigits in base b with $2 \le b \le 9$ are $2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88, 277,$ and 406. More precisely,*

$$\begin{aligned}
2 &= N_4 = \overline{10}_2, \\
3 &= N_5 = \overline{10}_3, \\
4 &= N_6 = \overline{100}_2 = \overline{10}_4, \\
6 &= N_7 = \overline{110}_2 = \overline{20}_3 = \overline{12}_4 = \overline{10}_6, \\
9 &= N_8 = \overline{100}_3 = \overline{21}_4 = \overline{14}_5 = \overline{13}_6 = \overline{12}_7 = \overline{10}_9, \\
13 &= N_9 = \overline{31}_4 = \overline{23}_5 = \overline{21}_6 = \overline{16}_7 = \overline{15}_8 = \overline{14}_9, \\
19 &= N_{10} = \overline{34}_5 = \overline{31}_6 = \overline{25}_7 = \overline{23}_8 = \overline{21}_9, \\
28 &= N_{11} = \overline{11100}_2 = \overline{40}_7 = \overline{34}_8 = \overline{31}_9, \\
41 &= N_{12} = \overline{1112}_3 = \overline{221}_4 = \overline{56}_7 = \overline{51}_8 = \overline{45}_9, \\
60 &= N_{13} = \overline{111100}_2 = \overline{330}_4 = \overline{220}_5 = \overline{114}_7 = \overline{74}_8, \\
88 &= N_{14} = \overline{224}_6, \\
277 &= N_{17} = \overline{544}_7 = \overline{337}_9, \ \textit{and} \\
406 &= N_{18} = \overline{3111}_5.
\end{aligned}$$

For the proof, we approach the standard procedure of obtaining bounds for certain linear forms in (nonzero) logarithms. The upper bounds are obtained via a manipulation of the associated Binet's formula for Narayana's sequence. For the lower bounds, we use the celebrated Baker's theorem on lower bounds for nonzero linear forms in logarithms of algebraic numbers due to Matveev. The bounds on the variables obtained via Baker's theorem are usually too large for computational purposes. To reduce the bounds, we use the Baker–Davenport reduction procedure.

2. Preliminaries

We prove the following lemma which gives a relation between n and $l+m$ of (1).

Lemma 1. *All solutions to Equation (1) satisfy*

$$(l+m-1)\log b + \log\alpha < n\log\alpha < (l+m)\log b + 1.$$

Proof. From (1) and (3), we get

$$\alpha^{n-2} \le N_n < b^{l+m}.$$

Taking the logarithm on both sides, we obtain

$$(n-2)\log\alpha < (l+m)\log b.$$

This leads to

$$n \log \alpha < (l+m) \log b + 2 \log \alpha < (l+m) \log b + 1.$$

On the other hand, for the lower bound, (1) implies

$$b^{l+m-1} < N_n \leq \alpha^{n-1}.$$

Taking the logarithm on both sides, we get

$$(l+m-1) \log b < (n-1) \log \alpha,$$

which implies

$$(l+m-1) \log b + \log \alpha < n \log \alpha.$$

$\square$

Baker's theory plays an important role in reducing the bounds concerning linear forms in logarithms of algebraic numbers. Let η be an algebraic number with minimal primitive polynomial

$$f(X) = a_0(X - \eta^{(1)}) \ldots (X - \eta^{(k)}) \in \mathbb{Z}[X],$$

where $a_0 > 0$, and $\eta^{(i)}$'s are conjugates of η. Then the *logarithmic height* of η is given by

$$h(\eta) = \frac{1}{k} \left(\log a_0 + \sum_{j=1}^{k} \max\{0, \log |\eta^{(j)}|\} \right).$$

In particular, if $\eta = a/b$ is a rational number with $\gcd(a, b) = 1$ and $b > 0$, then $h(\eta) = \log(\max\{|a|, b\})$. The following are some properties of the logarithmic height function:

$$\begin{aligned} h(\eta + \gamma) &\leq h(\eta) + h(\gamma) + \log 2, \\ h(\eta\gamma^{\pm 1}) &\leq h(\eta) + h(\gamma), \\ h(\eta^k) &= |k| h(\eta), \quad k \in \mathbb{Z}. \end{aligned}$$

With these notation, Matveev (see [17] or [8, Theorem 9.4]) proved the following result.

Theorem 2. *Let $\eta_1, \eta_2, \ldots, \eta_l$ be positive real algebraic numbers in a real algebraic number field $\mathbb{L}$ of degree $d_{\mathbb{L}}$ and $b_1, b_2, \ldots, b_l$ be non zero integers. If $\Gamma = \prod_{i=1}^{l} \eta_i^{b_i} - 1$ is not zero, then*

$$\log |\Gamma| > -1.4 \cdot 30^{l+3} l^{4.5} d_{\mathbb{L}}^2 (1 + \log d_{\mathbb{L}})(1 + \log D) A_1 A_2 \ldots A_l,$$

where $D = max\{|b_1|, |b_2|, \ldots, |b_l|\}$ and $A_1, A_2, \ldots, A_l$ are real numbers such that

$$A_j \geq max\{d_{\mathbb{L}} h(\eta_j), |\log \eta_j|, 0.16\} \quad \text{for } j = 1, \ldots, l.$$

We use the following reduction method of Baker-Davenport due to Dujella and Pethő [11, Lemma 5] for bound reduction.

Lemma 2. *Let M be a positive integer and p/q be a convergent of the continued fraction of the irrational number τ such that $q > 6M$. Let A, B, μ be real numbers with $A > 0$ and $B > 1$. Let $\varepsilon := \|\mu q\| - M\|\tau q\|$, where $\|.\|$ denotes the distance from the nearest integer. If $\varepsilon > 0$, then there exists no solution to the inequality*

$$0 < |u\tau - v + \mu| < AB^{-w},$$

in positive integers u, v, w with

$$u \leq M \text{ and } w \geq \frac{\log(Aq/\varepsilon)}{\log B}.$$

The following lemma will be used in our proof. It is seen in [15, Lemma 7].

Lemma 3. *Let $r \geq 1$ and $H > 0$ be such that $H > (4r^2)^r$ and $H > L/(\log L)^r$. Then*

$$L < 2^r H(\log H)^r.$$

3. Proof of Theorem 1

We are now able to prove Theorem 1.

Proof of Theorem 1. Assume that $n > 250$. Using Binet's formula of Narayana's cows sequence in (1), we get

$$C_\alpha \alpha^{n+2} + C_\beta \beta^{n+2} + C_\gamma \gamma^{n+2} = \frac{1}{b-1}\left(a_1 b^{l+m} - (a_1 - a_2)b^m - a_2\right). \tag{4}$$

We examine (4) in two different steps.

Firstly, we write (4) in the following way:

$$(b-1)C_\alpha \alpha^{n+2} - a_1 b^{l+m} = -(b-1)(C_\beta \beta^{n+2} + C_\gamma \gamma^{n+2}) - (a_1 - a_2)b^m - a_2.$$

Taking the absolute value on both sides and dividing by $a_1 b^{l+m}$, we get

$$\left|\left(\frac{(b-1)C_\alpha}{a_1}\right)\alpha^{n+2} b^{-(l+m)} - 1\right| < \frac{22 \cdot b^m}{a_1 b^{l+m}} < \frac{22}{b^l}. \tag{5}$$

Put

$$\Gamma = \left(\frac{(b-1)C_\alpha}{a_1}\right)\alpha^{n+2} b^{-(l+m)} - 1. \tag{6}$$

We need to show $\Gamma \neq 0$. If $\Gamma = 0$, then

$$C_\alpha \alpha^{n+2} = \frac{a_1}{b-1} b^{l+m}. \tag{7}$$

To show the above equality is absurd, let G be the Galois group of the splitting field of the characteristic polynomial $f(x)$ over $\mathbb{Q}$ and let $\sigma \in G$ be an automorphism such that $\sigma(\alpha) = \beta$. Applying σ on both sides of (7) and taking their absolute values, we get

$$|C_\beta \beta^{n+2}| = \frac{a_1}{b-1} b^{l+m}.$$

But, $|C_\beta \beta^{n+2}| < |C_\beta| = 0.407506\cdots < 1$, whereas $\frac{a_1}{b-1} b^{l+m} \geq 1$, which is not possible. Therefore, $\Gamma \neq 0$. To apply Theorem 2 in (6), let

$$\eta_1 = \frac{(b-1)C_\alpha}{a_1},\ \eta_2 = \alpha,\ \eta_3 = b,\ b_1 = 1,\ b_2 = n+2,\ b_3 = -(l+m),$$

where $\eta_1,\ \eta_2,\ \eta_3\ \in \mathbb{Q}(\alpha)$ and $b_1,\ b_2,\ b_3 \in \mathbb{Z}$. The degree $d_{\mathbb{L}} = [\mathbb{Q}(\alpha) : \mathbb{Q}]$ is 3. Since $l + m < n$, take $D = n$. The heights of η_1, η_2, η_3 are calculated as follows:

$$h(\eta_1) = h((b-1)C_\alpha/a_1) \leq h(8) + h(C_\alpha) + h(a_1) \leq 2\log 8 + \frac{\log 31}{3} < 5.4,$$

$$h(\eta_2) = h(\alpha) = \frac{\log \alpha}{3},\ h(\eta_3) = h(b) \leq \log 9.$$

Thus, we take

$$A_1 = 16.2,\ A_2 = \log \alpha,\ \text{and}\ A_3 = 3\log 9.$$

Applying Theorem 2, we find

$$\begin{aligned}\log|\Gamma| &> -1.4 \cdot 30^6 3^{4.5} 3^2 (1+\log 3)(1+\log(n+2))(16.2)(\log\alpha)(3\log 9)\\ &> -1.11 \cdot 10^{14} \log(1+\log(n+2)).\end{aligned}$$

Comparison of the above inequality with (5) gives

$$l \log b - \log 22 < 1.11 \cdot 10^{14}(1 + \log(n+2)),$$

which leads to

$$l \log b < 1.12 \cdot 10^{14}(1 + \log(n+2)). \tag{8}$$

Secondly, we rewrite (4) as

$$(b-1)C_\alpha \alpha^{n+2} - a_1 b^{l+m} + (a_1 - a_2)b^m = -(b-1)(C_\beta \beta^{n+2} + C_\gamma \gamma^{n+2}) - a_2.$$

Taking the absolute value on both sides and dividing by $(b-1)C_\alpha \alpha^{n+2}$, we obtain

$$\left| 1 - \left(\frac{a_1 b^l - (a_1 - a_2)}{(b-1)C_\alpha} \right) \alpha^{-(n+2)} b^m \right| < \frac{15}{(b-1)C_\alpha \alpha^{n+2}} < \frac{36}{\alpha^n}. \tag{9}$$

Put

$$\Gamma' = 1 - \left(\frac{a_1 b^l - (a_1 - a_2)}{(b-1)C_\alpha}\right) \alpha^{-(n+2)} b^m.$$

Using similar arguments as before, we can show that $\Gamma' \neq 0$. With the notation of Theorem 2, we take

$$\eta_1 = \frac{a_1 b^l - (a_1 - a_2)}{(b-1)C_\alpha},\ \eta_2 = \alpha,\ \eta_3 = b,\ b_1 = 1,\ b_2 = -(n+2),\ b_3 = m,$$

where $\eta_1,\ \eta_2,\ \eta_3\ \in \mathbb{Q}(\alpha)$ and $b_1,\ b_2,\ b_3 \in \mathbb{Z}$. The degree $d_{\mathbb{L}} = [\mathbb{Q}(\alpha) : \mathbb{Q}]$ is 3. Since $m < n+2$, $D = n+2$. Computing the logarithmic heights of η_1, η_2 and η_3, we get

$$h(\eta_2) = \frac{\log \alpha}{3},\ h(\eta_3) = \log b$$

and

$$\begin{aligned}
h(\eta_1) &\leq h(a_1 b^l - (a_1 - a_2)) + h((b-1)C_\alpha) \\
&\leq h(a_1) + l h(b) + h(a_1 - a_2) + h(b-1) + h(C_\alpha) + \log 2 \\
&< 4\log 8 + 2\log 2 + \frac{\log 31}{3} + l \log b \\
&< 10.85 + l \log b.
\end{aligned}$$

Hence, from (8), we get

$$h(\eta_1) < 10.85 + 1.12 \cdot 10^{14}(1 + \log(n+2)).$$

So, we take

$$A_1 = 3.39 \cdot 10^{14}(1 + \log(n+2)),\ A_2 = \log \alpha,\ \text{and}\ A_3 = 3\log 9.$$

Using all these values in Theorem 2, we have

$$\begin{aligned}
\log |\Gamma'| > -1.4 \cdot 30^6 3^{4.5} 3^2 (1 + \log 3)(1 + \log(n+2))(3.39 \cdot 10^{14}(1 + \log(n+2))) \\
\cdot (\log \alpha)(3 \log 9).
\end{aligned}$$

Comparing the above inequality with (9), we obtain

$$n \log \alpha - \log(36) < 2.32 \cdot 10^{27}(1 + \log(n+2))^2.$$

Thus, we conclude that

$$n < 6.13 \cdot 10^{27}(1 + \log(n+2))^2 < 9.8 \cdot 10^{28}(\log n)^2.$$

With the notation of Lemma 3, we take $r = 2$, $L = n$, and $H = 9.8 \cdot 10^{28}$. Applying Lemma 3, we have

$$\begin{aligned} n &< 2^2(9.8 \cdot 10^{28})(\log(9.8 \cdot 10^{28}))^2 \\ &< 1.75 \cdot 10^{33}. \end{aligned}$$

Our next aim is to reduce these bounds of (1). Put

$$\Lambda = (l+m)\log b - (n+2)\log\alpha - \log\left(\frac{C_\alpha(b-1)}{a_1}\right).$$

Then (5) can be written as

$$|e^{-\Lambda} - 1| < \frac{22}{b^l}.$$

Observe that $\Lambda \neq 0$ as $e^{-\Lambda} - 1 = \Gamma \neq 0$. Assuming $l \geq 6$, the right-hand side in the above inequality is at most $\frac{22}{64} < \frac{1}{2}$. The inequality $|e^z - 1| < y$ for real values of z and y implies $|z| < 2y$. Thus, we get

$$|\Lambda| < \frac{44}{b^l},$$

which implies that

$$\left|(l+m)\log b - n\log\alpha - \log\left(\frac{\alpha^2 C_\alpha(b-1)}{a_1}\right)\right| < \frac{44}{b^l}.$$

Dividing both sides by $\log\alpha$, we get

$$\left|(l+m)\left(\frac{\log b}{\log\alpha}\right) - n - \left(\frac{\log(\alpha^2 C_\alpha(b-1)/a_1)}{\log\alpha}\right)\right| < \frac{116}{b^l}. \tag{10}$$

To apply Lemma 2 in (10), let

$$u = l+m,\ \tau = \left(\frac{\log b}{\log\alpha}\right),\ v = n,\ \mu = -\left(\frac{\log(\alpha^2 C_\alpha(b-1)/a_1)}{\log\alpha}\right),$$
$$A = 116,\ B = b,\ w = l.$$

Choose $M = 1.75 \cdot 10^{33}$. Applying Lemma 2, we find the following results given in Table 1.

b	2	3	4	5	6	7	8	9
q_t	q_{79}	q_{62}	q_{74}	q_{69}	q_{58}	q_{63}	q_{72}	q_{64}
$\epsilon \geq$	0.085	0.103	0.189	0.187	0.097	0.126	0.030	0.007
$l \leq$	123	77	61	52	47	44	42	40

Table 1: Bounds on l

Thus, in all cases $l \leq 123$. Now, for $1 \leq a_1, a_2 \leq 8$ and $l \leq 123$, put

$$\Lambda' = m \log b - (n+2) \log \alpha + \log \left(\frac{a_1 b^l - (a_1 - a_2)}{(b-1)C_\alpha} \right).$$

Then (9) can be written as

$$|e^{-\Lambda'} - 1| < \frac{36}{\alpha^n}.$$

Observe that $\Lambda' \neq 0$ as $e^{-\Lambda'} - 1 = \Gamma' \neq 0$. Since $n > 250$, the right-hand side in the above inequality is at most $\frac{36}{\alpha^{251}} < \frac{1}{2}$. The inequality $|e^z - 1| < y$ for real values of z and y implies $|z| < 2y$. Thus, we get

$$|\Lambda'| < \frac{72}{\alpha^n},$$

which implies that

$$\left| m \log b - n \log \alpha + \log \left(\frac{a_1 b^l - (a_1 - a_2)}{(b-1)C_\alpha \alpha^2} \right) \right| < \frac{72}{\alpha^n}.$$

Dividing both sides by $\log \alpha$, we get

$$\left| m \left(\frac{\log b}{\log \alpha} \right) - n + \left(\frac{\log(a_1 b^l - (a_1 - a_2)/((b-1)C_\alpha \alpha^2)}{\log \alpha} \right) \right| < \frac{188.4}{\alpha^n}. \tag{11}$$

To apply Lemma 2 in (11), let

$$u = m, \ \tau = \left(\frac{\log b}{\log \alpha} \right), \ v = n, \ \mu = \left(\frac{\log(a_1 b^l - (a_1 - a_2)/((b-1)C_\alpha \alpha^2)}{\log \alpha} \right),$$
$$A = 188.4, \ B = \alpha, \ w = n.$$

Choose $M = 1.75 \cdot 10^{33}$. With the help of *Mathematica*, we find the following results given in Table 2.

b	2	3	4	5	6	7	8	9
q_t	q_{81}	q_{63}	q_{74}	q_{69}	q_{58}	q_{63}	q_{72}	q_{64}
$\epsilon \geq$	0.00295	0.00061	0.00065	0.00054	0.00002	0.00012	0.00023	0.00021
$n \leq$	237	241	238	239	248	244	243	241

Table 2: Bounds on n

Thus, $n \leq 248$ in all cases, which is a contradiction to our assumption that $n > 250$. This completes the proof of the theorem. □

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