2.1 The tangent and Velocity Problems

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09/01/2010

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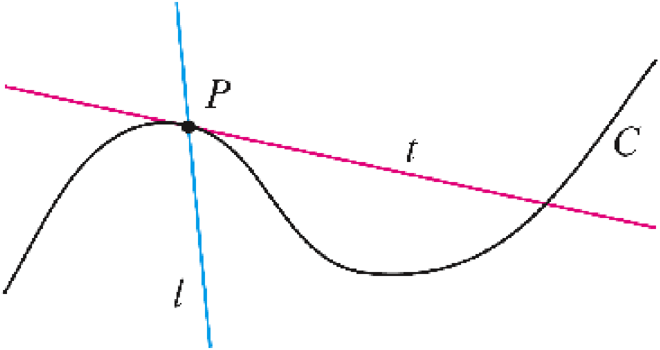

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Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

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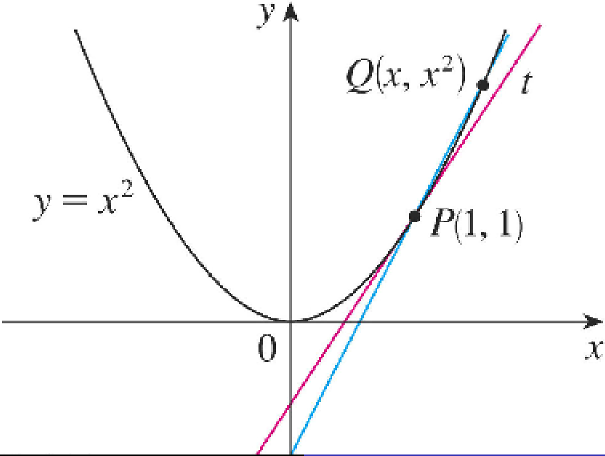

Example (cont'd)

x	m_{PQ}
2	3
1.5	2.5
1.1	2.1
1.01	2.01
1.001	2.001

x	m_{PQ}
0	1
0.5	1.5
0.9	1.9
0.99	1.99
0.999	1.999

Falling Ball

time (s)	distance (m)
0.10	0.049
0.20	0.196
0.30	0.441
0.40	0.784
0.50	1.225
0.60	1.764
0.70	2.401
0.80	3.136
0.90	3.969
1.00	4.900

Average Speed

Fact

If the distance fallen after t seconds is denoted by s(t) and measured in meters, then Galileo's law is expressed by the following equation.

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Definition

Average speed is defined to be **change in distance divided by change in time.**

Derived Table of the average speed $[t, t+0.1]$

time (s)	distance (m)	speed (m/s)
0.10	0.049	1.470000
0.20	0.196	2.450000
0.30	0.441	3.430000
0.40	0.784	4.410000
0.50	1.225	5.390000
0.60	1.764	6.370000
0.70	2.401	7.350000
0.80	3.136	8.330000
0.90	3.969	9.310000
1.00	4.900	10.290000

Instantaneous velocity

Definition

The instantaneous speed is *a limiting value* of the average speeds as the interval h between successive times shrinks to zero.

$$v(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}.$$

Approximating instantaneous velocity for $t = 0.5$

h	$\frac{s(t+h)-s(t)}{h}$
0.1	5.39
0.05	5.145
0.02	4.998
0.01	4.949
0.001	4.9005
0.0001	4.9000003