3.1 Derivatives of polynomials and exponential functions

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Building the Toolbox

- Derivative of a constant

$$\frac{d}{dx}(c) = 0$$

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- Derivative of a constant

$$\frac{d}{dx}(c) = 0$$

- The power rule

$$\frac{d}{dx}(x^r) = rx^{r-1},$$

  where r is any real number.

Example

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Find the derivative of the following functions:

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- $f(x) = \sqrt{50}$

Example

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- $f(x) = \sqrt{50}$
- $f(x) = \sqrt[5]{x^6}$

Building the Toolbox ...

- Derivative of the natural exponential function

$$\frac{d}{dx}(e^x) = e^x.$$

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- Derivative of the natural exponential function

$$\frac{d}{dx}(e^x) = e^x.$$

- Derivative of any exponential function

$$\frac{d}{dx}(a^x) = a^x \ln a.$$

The constant, sum, and differentiation rules

- The constant multiple rule: If c is a constant then

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- The difference rule:

$$\frac{d}{dx}\left[f(x)-g(x)\right] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x).$$

Examples

Example

Find

Examples

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Find

- $\frac{d}{dx}(3x^2 + 2x + 7)$

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- $\frac{d}{dx}(x + \sqrt{x})$

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Find

- $\frac{d}{dx}(3x^2 + 2x + 7)$
- $\frac{d}{dx}(x + \sqrt{x})$
- $\frac{d}{dx}\left(2e^x + \frac{3}{x} + \frac{4}{x^2}\right)$.

3.2 The Product Rule

Theorem

If $f(x)$ and $g(x)$ are functions with derivatives $f'(x)$ and $g'(x)$, respectively, then

$$(fg)'(x) = f(x)g'(x) + g(x)f'(x).$$

In words, "the derivative of a product is the first factor times the derivative of the second, plus the second factor times the derivative of the first".

Example

Examples

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- Find $f'(x)$ in two ways, given $f(x) = (5x+3)(x+2)$.

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- If $y = \sqrt{x}(x^2 + 2)$, find $\frac{dy}{dx}$.

The Reciprocal Rule

Theorem

Suppose f *has derivative* f'*. Then for any* x *such that* $f(x) \neq 0$,
$\left(\frac{1}{f}\right)' = -\frac{f(x)'}{f(x)^2}$. *That is,* $\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$.

Example

Example

- Find $f'(x)$ given $f(x) = \frac{1}{x^2+1}$.

The Quotient Rule

Theorem

Suppose f and g have derivatives f' and g', respectively. Then for any x such that $g(x) \neq 0$,

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f(x)' - f(x)g(x)'}{g(x)^2}.$$

That is,

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}.$$

In words, "the derivative of a quotient is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator all divided by the denominator squared".

Examples

Example

Example

- Find $f'(x)$ given

$$f(x) = \frac{x+1}{x+2}.$$

Example

- Find $f'(x)$ given
$$f(x) = \frac{x+1}{x+2}.$$
- Find $f'(x)$ given
$$f(x) = \frac{1+\sqrt{x}}{x^2+3x+2}.$$

Example

Example

- For $f(x) = \frac{1}{x} = x^{-1}$, find the derivative three ways, using the power rule, the reciprocal rule, and the quotient rule.