3.1 Derivatives of polynomials and exponential functions

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- Derivative of a constant

$$\frac{d}{dx}(c) = 0$$

- The power rule

$$\frac{d}{dx}(x^r) = rx^{r-1},$$

where r is any real number.

Example

Example

Find the derivative of the following functions:
- $f(x) = \sqrt{50}$
- $f(x) = \sqrt[5]{x^6}$

- Derivative of the natural exponential function

$$\frac{d}{dx}(e^x) = e^x.$$

- Derivative of any exponential function

$$\frac{d}{dx}(a^x) = a^x \ln a.$$

- The constant multiple rule: If c is a constant then

$$\frac{d}{dx}\big[cf(x)\big] = c\frac{d}{dx}f(x).$$

- The sum rule:

$$\frac{d}{dx}\big[f(x) + g(x)\big] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x).$$

- The difference rule:

$$\frac{d}{dx}\big[f(x) - g(x)\big] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x).$$

Examples

Example

Find

- $\frac{d}{dx}\left(3x^2 + 2x + 7\right)$
- $\frac{d}{dx}\left(x + \sqrt{x}\right)$
- $\frac{d}{dx}\left(2e^x + \frac{3}{x} + \frac{4}{x^2}\right).$

Theorem

If $f(x)$ and $g(x)$ are functions with derivatives $f'(x)$ and $g'(x)$, respectively, then

$$(fg)'(x) = f(x)g'(x) + g(x)f'(x).$$

In words, "the derivative of a product is the first factor times the derivative of the second, plus the second factor times the derivative of the first".

Example

- Find $f'(x)$ in two ways, given $f(x) = (5x + 3)(x + 2)$.
- If $y = \sqrt{x}(x^2 + 2)$, find $\frac{dy}{dx}$.

Theorem

Suppose f has derivative f'. Then for any x such that $f(x) \neq 0$, $\left(\frac{1}{f}\right)' = -\frac{f(x)'}{f(x)^2}$. That is, $\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$.

Example

Example

- Find $f'(x)$ given $f(x) = \frac{1}{x^2+1}$.

Theorem

Suppose f and g have derivatives f' and g', respectively. Then for any x such that $g(x) \neq 0$,

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f(x)' - f(x)g(x)'}{g(x)^2}.$$

That is,

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}.$$

In words, "the derivative of a quotient is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator all divided by the denominator squared".

Example

- Find $f'(x)$ given
$$f(x) = \frac{x+1}{x+2}.$$

- Find $f'(x)$ given
$$f(x) = \frac{1 + \sqrt{x}}{x^2 + 3x + 2}.$$

Example

- For $f(x) = \frac{1}{x} = x^{-1}$, find the derivative three ways, using the power rule, the reciprocal rule, and the quotient rule.