3.4 The Chain Rule

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09/26/2010

Theorem

Let $(f \circ g)(x) = f(g(x))$ be the function defined from f and g by composition. Assume that g is differentiable at the point x and that f is differentiable at the point $g(x)$. Then the composite function $f \circ g$ is differentiable at the point x, and

$$(f \circ g)'(x) = [f(g(x))]' = f'(g(x))g'(x)$$

Using Leibniz's notation:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Examples

Example

- Differentiate
$$f(x) = \sqrt{x^2 + 1}.$$
- Differentiate
$$y = (x^2 + 2)^{10}.$$

Examples

Example

- Differentiate
$$f(x) = (1 + 3\sqrt{x})^{35}.$$
- Differentiate
$$f(x) = \left(\frac{x+1}{x^2+1}\right)^3.$$
- Differentiate
$$y = e^{\frac{1}{x}}.$$
- Differentiate
$$y = 10^{1-x^2}.$$