

Derivatives of the Trigonometric Functions

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The Derivative of sin

$$\begin{aligned}\frac{d}{dx} \sin x &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \\ &= \sin x \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} + \cos x \lim_{h \rightarrow 0} \frac{\sin h}{h}\end{aligned}$$

Theorem

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1,$$

and

$$\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0.$$

Example

Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$$

Theorem

The derivatives of $\sin x$ and $\cos x$ are given by

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x.$$

Example

- Differentiate $\sin(2x)$.
- Differentiate $f(x) = \sin\left(x^2 + \frac{1}{x}\right)$.
- Differentiate $y = \cos(3x + \sqrt{x})$.

Example

- Differentiate

$$y = \sin x \cos x$$

- Differentiate

$$y = \sin^2(\cos(x^2 + 2))$$

Theorem

Derivatives of other trigonometric functions

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

Example

Example

Find

$$\frac{d}{dx} \sec^2 x$$

Example

Compute the derivative of

$$y = (\sin^3(\tan^2(2x)))^4.$$