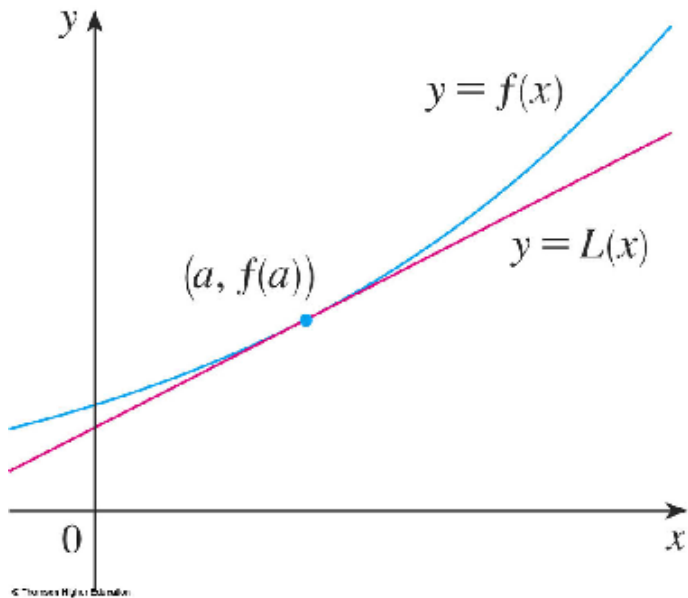


3.10 Linear approximations and Differentials

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Linear approximations



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is called *the linear approximation* or *tangent line approximation* of f at a .

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- The function whose graph is the tangent line

$$L(x) = f(a) + f'(a)(x - a)$$

is called the **linearization** of f at a .

Example

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Find the linearization of $y = \sqrt{x + 1}$ at $a = 1$ and use it to approximate the numbers $\sqrt{0.98}$ and $\sqrt{1.02}$.

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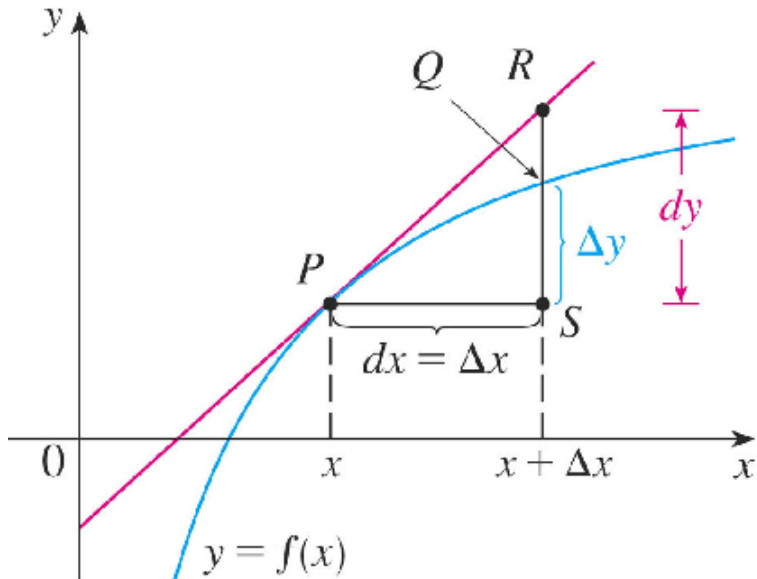
- Recall that if Δx is the change in x then the change in y is

$$\Delta y = f(x + \Delta x) - f(x).$$

- The **differential** dx is an independent variable.
- The **differential** dy is

$$dy = f'(x)dx.$$

Differentials: graph



Example

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- $y = \ln \sqrt{1 + x^2}$

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- $y = \frac{1}{x+1}$, $x = 1$, $dx = -0.01$.
- $y = \sqrt{x}$, $x = 1$, $dx = 0.1$.