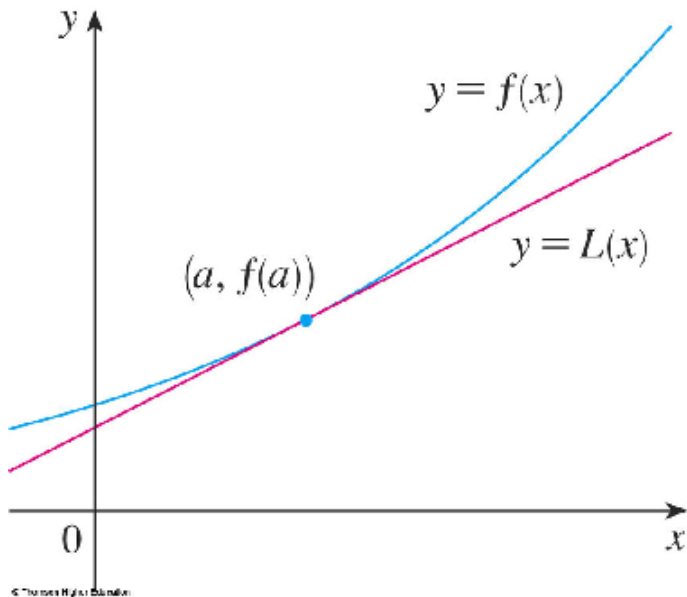


3.10 Linear approximations and Differentials

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Linear approximations



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Definition

- The approximation

$$f(x) \approx f(a) + f'(a)(x - a)$$

is called ***the linear approximation*** or ***tangent line approximation*** of f at a .

- The function whose graph is the tangent line

$$L(x) = f(a) + f'(a)(x - a)$$

is called the **linearization** of f at a .

Example

Example

Find the linearization of $y = \sqrt{x + 1}$ at $a = 1$ and use it to approximate the numbers $\sqrt{0.98}$ and $\sqrt{1.02}$.

Definition

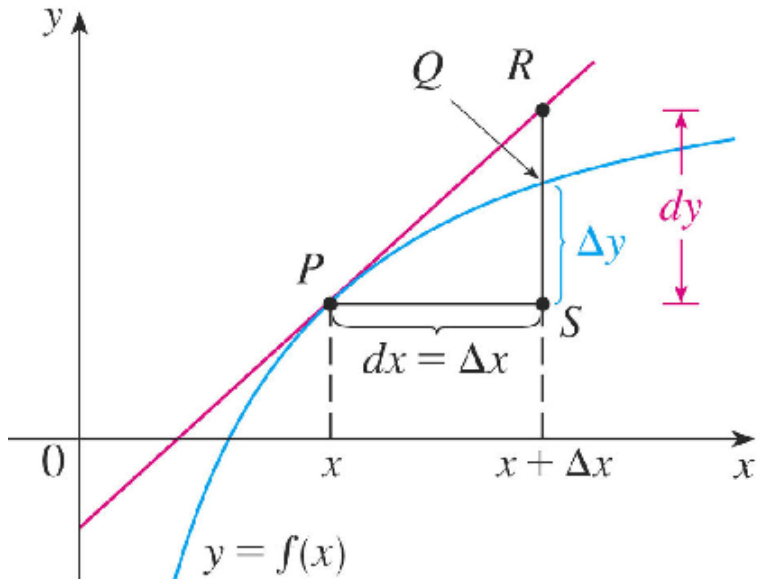
- Recall that if Δx is the change in x then the change in y is

$$\Delta y = f(x + \Delta x) - f(x).$$

- The **differential** dx is an independent variable.
- The **differential** dy is

$$dy = f'(x)dx.$$

Differentials: graph



Example

Find the differential of the following functions

- $y = x^2 \cos(x)$
- $y = \ln \sqrt{1 + x^2}$

Example

Find the differential dy , evaluate dy for the given values of x and dx , and compare it to Δy :

- $y = \frac{1}{x+1}$, $x = 1$, $dx = -0.01$.
- $y = \sqrt{x}$, $x = 1$, $dx = 0.1$.