4.1 Maximum and Minimum Values

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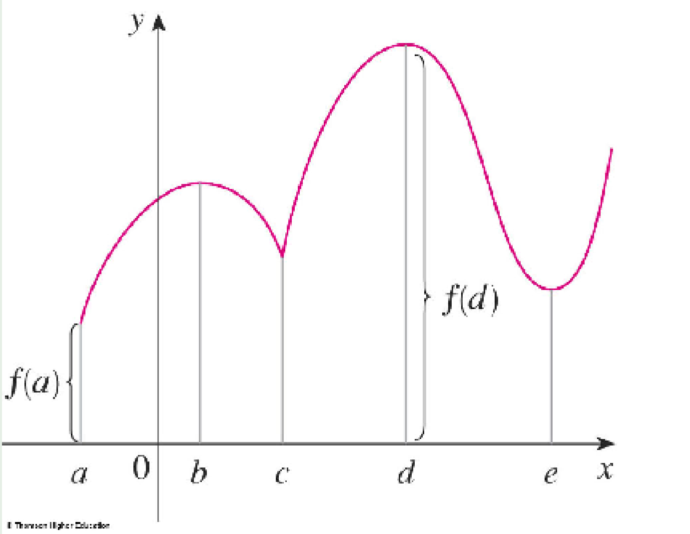
10/27/2010

Extreme Values

Definition

- A function f has an absolute maximum (or global maximum) at c if $f(c) \geq f(x)$ for all x in D, where D is the domain of f.
- The number $f(c)$ is called the maximum value of f on D.
- f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in D and the number $f(c)$ is called the minimum value of f on D.
- The maximum and minimum values of f are called the extreme values of f.

Example



Local maximum and local minimum

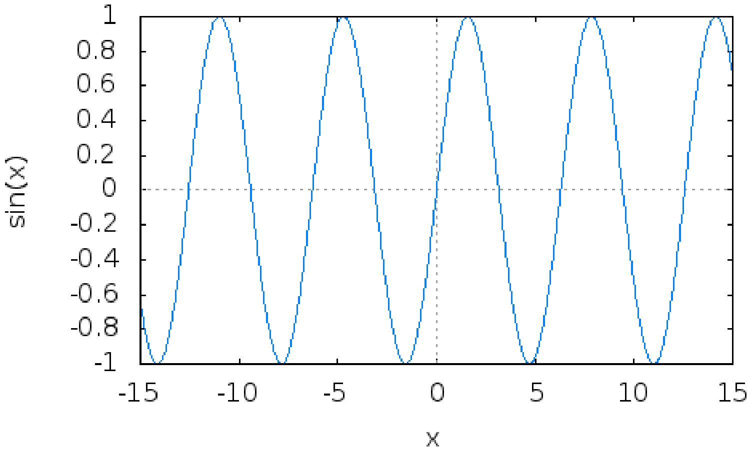
Definition

- A function f has a local maximum (or relative maximum) at c if $f(c) \geq f(x)$ when x is near c.
- f has a local minimum at c if $f(c) \leq f(x)$ when x is near c.

Example

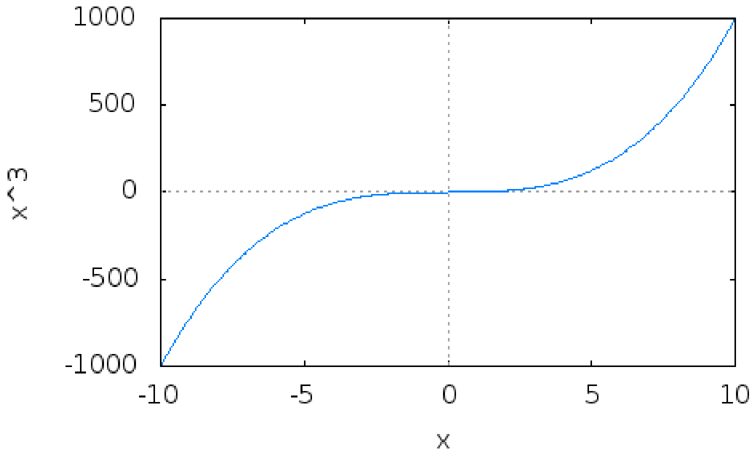
Example

Let $f(x) = \sin x$



Example

Let $f(x) = x^3$



The Extreme Value Theorem

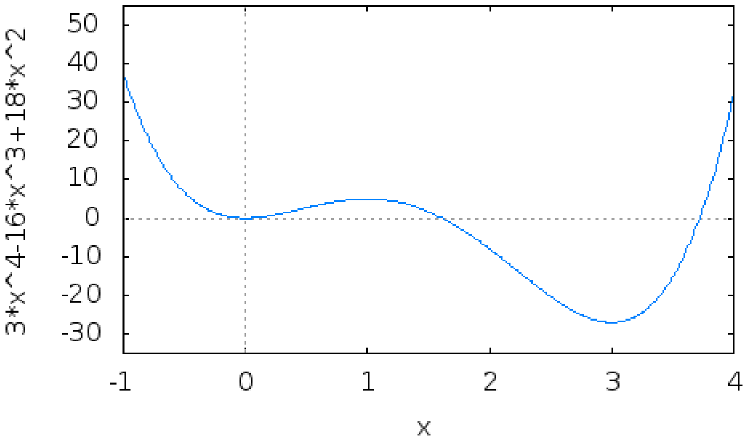
Fact

If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Example

Example

Let $f(x) = 3x^4 - 16x^3 + 18x^2$ with $-1 \le x \le 4$.



Fermat's Theorem

Fact

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

Example

Example

- Let $f(x) = |x|$. What does the Fermat's theorem say for this function?
- What about $f(x) = x^3$?

Critical Numbers

Definition

A critical number of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Examples

Example

Find the critical points of the following functions:

- $f(x) = x^3 + 3x^2 - 24x$
- $f(x) = \frac{x-1}{x^2-x+1}$
- $f(x) = x^{-2} \ln x$

Fact

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. *Find the values of f at the critical numbers of f in (a, b).*
2. *Find the values of f at the endpoints of the interval.*
3. *The largest value from 1 and 2 is the absolute maximum value.*
4. *The smallest is the absolute minimum value.*

Example

Find the absolute maximum and absolute minimum values of f on the given interval

- $f(x) = x^3 - 3x + 1$, $[0, 4]$.
- $f(x) = (x^2 - 1)^3$, $[-1, 2]$.
- $f(x) = x - \ln x$, $[1/2, 2]$.
- $f(x) = e^{-x} - e^{-2x}$, $[0, 1]$.