4.2 The Mean Value Theorem

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Rolle's Theorem

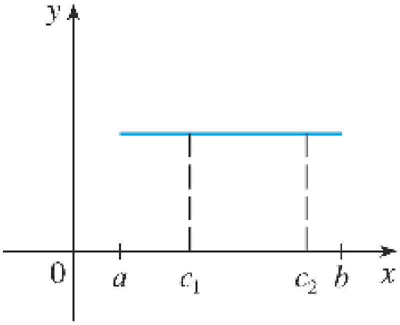
Fact

Let f be a function that satisfies the following three hypotheses:

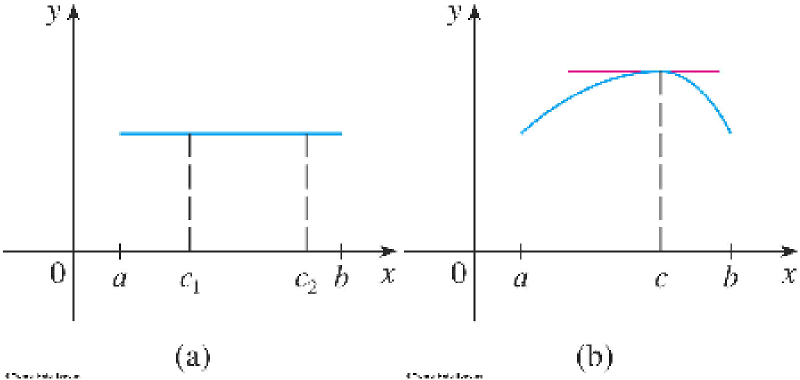
1. f *is continuous on the closed interval* $[a, b]$.
2. f *is differentiable on the open interval* (a, b).
3. $f(a) = f(b)$

Then, there is a number c in (a, b) such that $f'(c) = 0$.

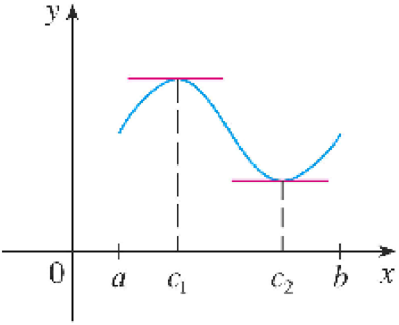
Examples



(a)

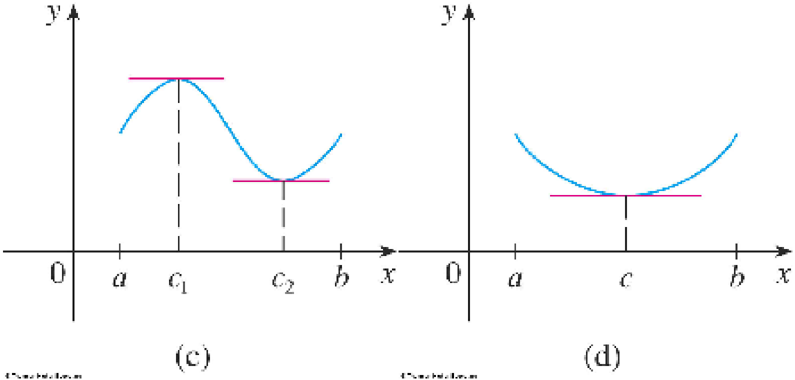

y
0
a
c_1
c_2
b
x
(a)
y
0
a
c
b
x
(b)

Examples



(c)

Examples



Example

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- Prove that the equation $x^3 + x - 1 = 0$ has exactly one real root.

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- Prove that the equation $x^3 + x - 1 = 0$ has exactly one real root.
- Show that the equation $2x - 1 - \sin x = 0$ has exactly one real root.

The Mean Value Theorem

Fact

Let f be a function that fulfills two hypotheses:

1. *f is continuous on the closed interval $[a, b]$.*
2. *f is differentiable on the open interval (a, b).*

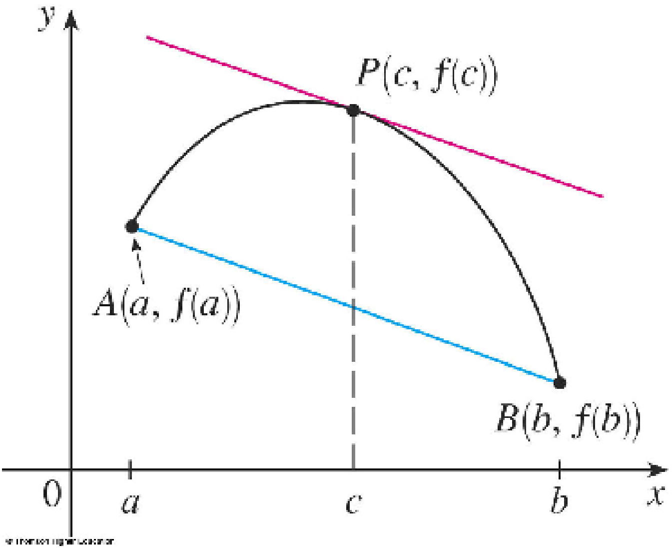
Then, there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

$$f(b) - f(a) = f'(c)(b - a).$$

Example



Example

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- $f(x) = x^3 + x - 1$, $[0, 2]$.
- $f(x) = e^{-2x}$, $[0, 3]$.

Example

Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x. How large can $f(2)$ possibly be?