2.1 The tangent and Velocity Problems
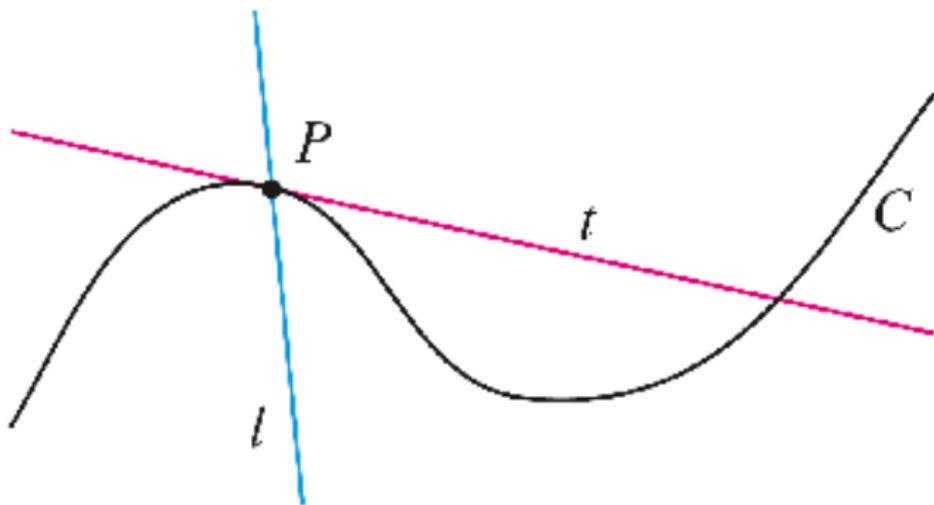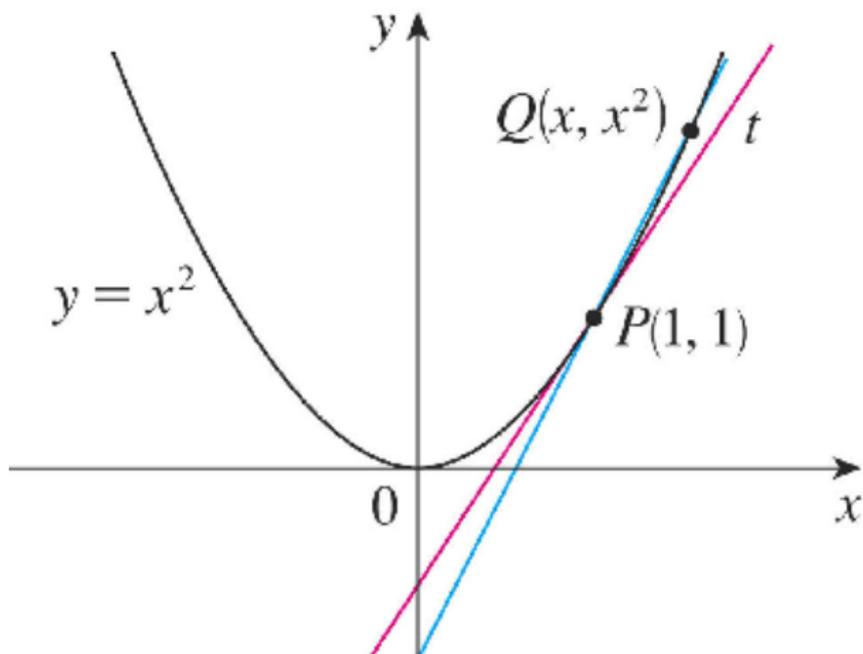
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- The word tangent is derived from the Latin word tangens, which means 'touching.'
- Thus, a tangent to a curve is a line that touches the curve.

Example

Example

Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

Example (cont'd)

x	m_{PQ}
2	3
1.5	2.5
1.1	2.1
1.01	2.01
1.001	2.001

x	m_{PQ}
0	1
0.5	1.5
0.9	1.9
0.99	1.99
0.999	1.999

Falling Ball

time (s)	distance (m)
0.10	0.049
0.20	0.196
0.30	0.441
0.40	0.784
0.50	1.225
0.60	1.764
0.70	2.401
0.80	3.136
0.90	3.969
1.00	4.900

Fact

If the distance fallen after t seconds is denoted by s(t) and measured in meters, then Galileo's law is expressed by the following equation.

$$s(t) = 4.9t^2$$

Definition

Average speed is defined to be **change in distance divided by change in time.**

time (s)	distance (m)	speed (m/s)
0.10	0.049	1.470000
0.20	0.196	2.450000
0.30	0.441	3.430000
0.40	0.784	4.410000
0.50	1.225	5.390000
0.60	1.764	6.370000
0.70	2.401	7.350000
0.80	3.136	8.330000
0.90	3.969	9.310000
1.00	4.900	10.290000

Definition

The instantaneous speed is *a limiting value* of the average speeds as the interval h between successive times shrinks to zero.

$$v(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}.$$

h	$\frac{s(t+h)-s(t)}{h}$
0.1	5.39
0.05	5.145
0.02	4.998
0.01	4.949
0.001	4.9005
0.0001	4.9000003