4.4 Indeterminate Forms and L'Hospital Rule

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Example

Consider the following limits

- $\lim_{x\to 1} \frac{x-1}{x^2-1}$
- $\lim_{x\to 0} \frac{\sin x}{x}$
- $\lim_{x\to 1} \frac{\ln x}{x-1}$

Indeterminate Forms

Definition

If we have a limit of the form

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

where both $f(x) \to 0$ and $g(x) \to 0$ as $x \to a$, then this limit may or may not exist.
It is called an indeterminate form of type

$$\frac{0}{0}.$$

If both $f(x) \to \infty$ and $g(x) \to \infty$ as $x \to a$, then the limit is called an indeterminate form of the type

$$\frac{\infty}{\infty}.$$

L'Hospital Rule

Fact

Suppose f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a).
Suppose that

$$\lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to \infty} g(x) = 0$$

or that

$$\lim_{x \to a} f(x) = \infty \quad \text{and} \quad \lim_{x \to \infty} g(x) = \infty$$

Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)},$$

if the limit on the right exists.

Example

Find the following limits

- $\lim_{x\to\infty} \frac{e^x}{x^2}$
- $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$
- $\lim_{x\to\pi^-} \frac{\sin x}{1-\cos x}$.
- $\lim_{x\to\infty} \frac{\ln\ln x}{x}$
- $\lim_{x\to 1} \frac{\ln x}{\sin \pi x}$
- $\lim_{x\to-\infty} x^2 e^x$
- $\lim_{x\to\infty} x\tan(1/x)$

Examples

Example

Find the following limits

- $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$
- $\lim_{x\to 0}(\csc x-\cot x)$
- $\lim_{x\to 0^+}(\tan 2x)^x$.