4.5 Summary of Curve Sketching

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  - periodic function: if $f(x) = f(x + p)$ for all x (where p is a fixed number) then f is a *periodic function*.

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  - **Vertical asymptotes:** The line $x = a$ is a vertical asymptote for the curve $y = f(x)$ if one of the following is true

$$\begin{aligned} \lim_{x\to a^-} f(x) &= \infty \qquad \lim_{x\to a^-} f(x) = -\infty \\ \lim_{x\to a^+} f(x) &= \infty \qquad \lim_{x\to a^+} f(x) = -\infty \end{aligned}$$

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  - If f' changes from positive to negative at a critical number c, then $f(c)$ is a local maximum.
  - If f' changes from negative to positive at c, then $f(c)$ is a local minimum.

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  - Plot the intercepts, maximum and minimum points, and inflection points.

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- **Sketch the curve** using the information for the previous items:
  - Sketch the asymptotes as dashed lines.
  - Plot the intercepts, maximum and minimum points, and inflection points.
  - Then, make the curve pass through these points

Example

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Sketch the graph of

$$\frac{x^2}{\sqrt{x+1}}.$$

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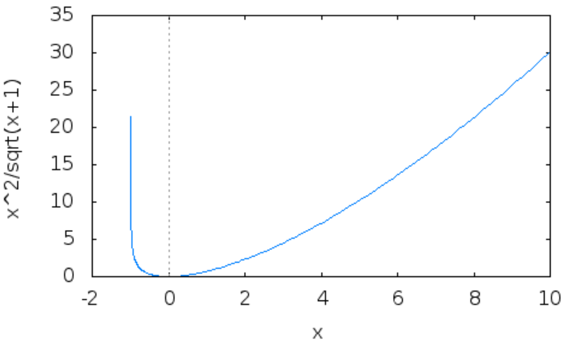

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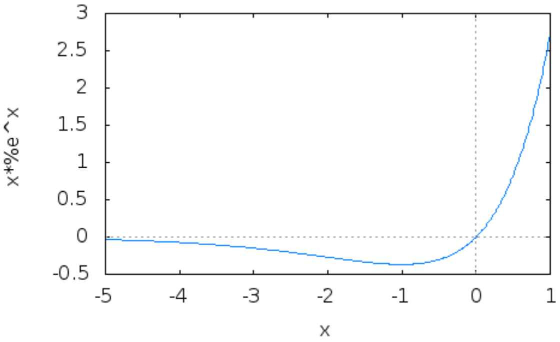

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$$f(x) = 8x^2 - x^4.$$

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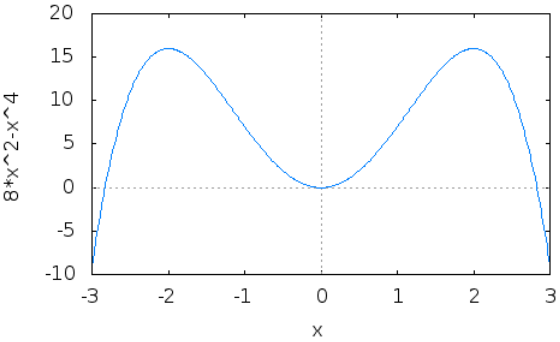

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$$f(x) = \frac{x}{x^3 - 1}$$

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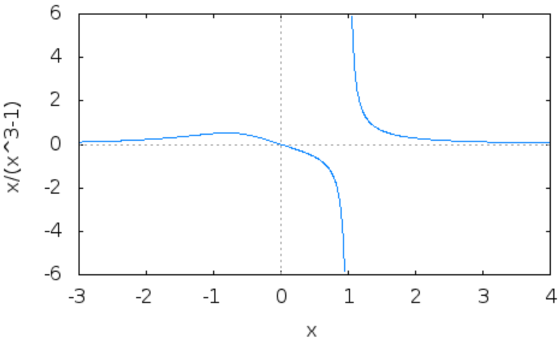

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$$f(x) = \sqrt{x^2 + x} - x.$$

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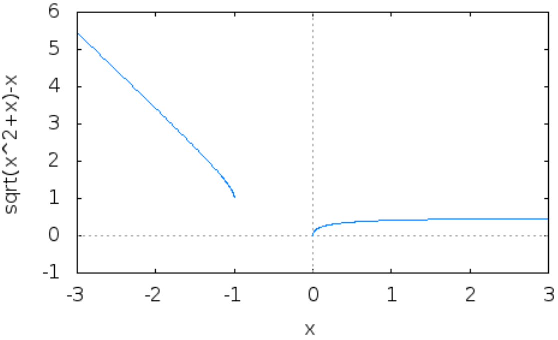

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$$f(x) = \frac{\sin x}{2 + \cos x}.$$

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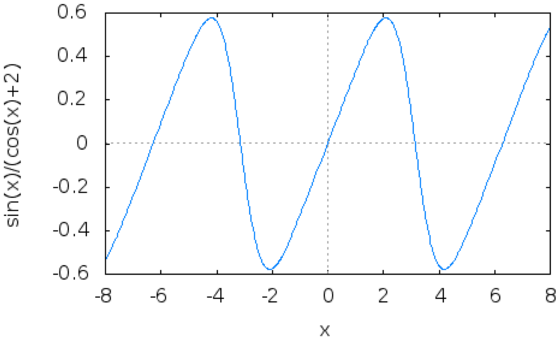

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$$f(x) = e^{2x} - e^{x}.$$

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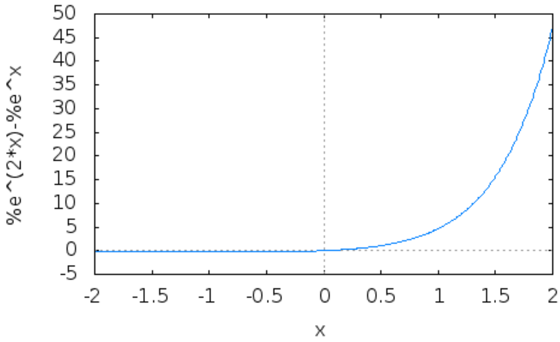

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