4.5 Summary of Curve Sketching

Marius Ionescu

11/17/2010

Guidelines for sketching a curve

- **Domain**: the set of values of x for which $f(x)$ is defined.
- **Intercepts:** The y-intercept is $f(0)$ and this tells us where the curve intersects the y-axis.
  To find the x-intercepts, we set $y = 0$ and solve for x (if possible).
- **Symmetry:**
  - even functions: if $f(x) = f(-x)$ then f is an *even function* and the curve is symmetric about the y-axis.
  - odd functions: if $f(x) = -f(-x)$ then f is an *odd function* and the curve is symmetric about the origin.
  - periodic function: if $f(x) = f(x + p)$ for all x (where p is a fixed number) then f is a *periodic function*.

Guidelines for sketching a curve

- **Asymptotes:**
  - **Horizontal asymptotes:** If $\lim_{x\to-\infty} f(x) = L$ or $\lim_{x\to\infty} f(x) = L$ then the line $y = L$ is a horizontal asymptote of the curve $y = f(x)$.
  - **Vertical asymptotes:** The line $x = a$ is a vertical asymptote for the curve $y = f(x)$ if one of the following is true

$$\lim_{x\to a^-} f(x) = \infty \qquad \lim_{x\to a^-} f(x) = -\infty$$
$$\lim_{x\to a^+} f(x) = \infty \qquad \lim_{x\to a^+} f(x) = -\infty$$

Guidelines for sketching a curve

- **Intervals of Increase of Decrease:** use the first derivative test - Compute $f'(x)$ and find the intervals on which:
  - $f'(x)$ is positive (f is increasing).
  - $f'(x)$ is negative (f is decreasing).
- **Local Maximum and Minimum Values:** Find the critical numbers of f (the numbers c where $f'(c) = 0$ or $f'(c)$ does not exist). Then, use the First Derivative Test.
  - If f' changes from positive to negative at a critical number c, then $f(c)$ is a local maximum.
  - If f' changes from negative to positive at c, then $f(c)$ is a local minimum.

Guidelines for sketching a curve

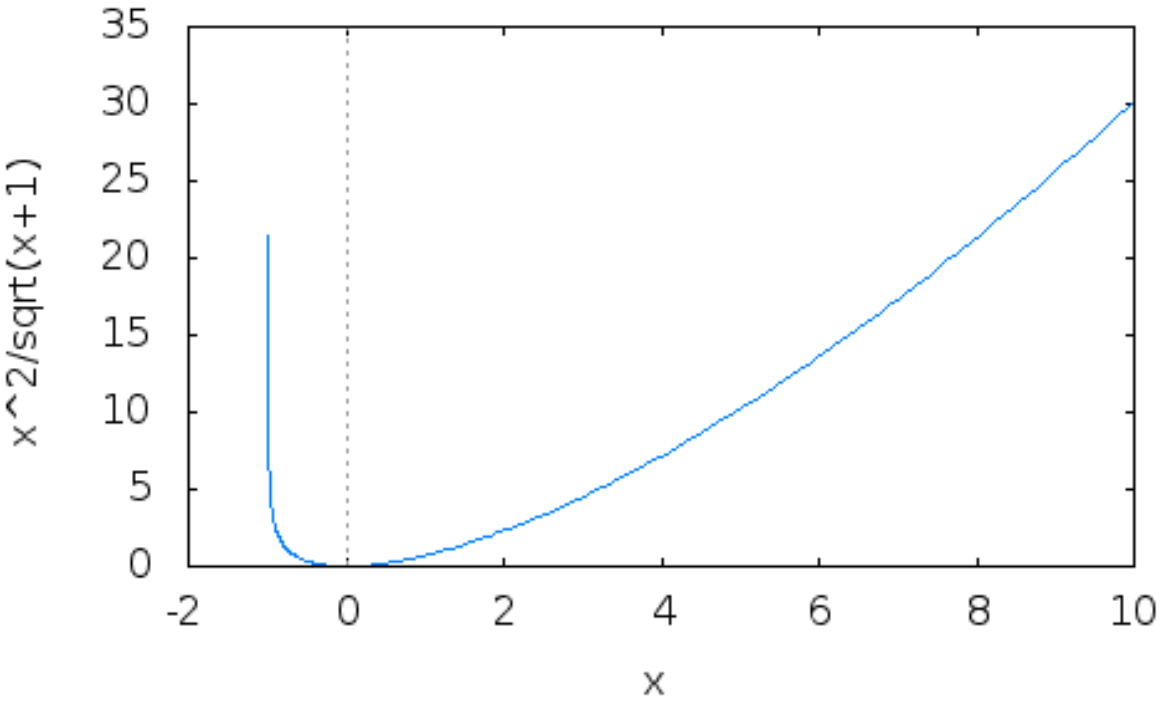
- **Concavity and Points of Inflection:** Compute $f''(x)$ and use the Concavity Test. The curve is:
  - Concave upward where $f''(x) > 0$
  - Concave downward where $f''(x) < 0$
- **Sketch the curve** using the information for the previous items:
  - Sketch the asymptotes as dashed lines.
  - Plot the intercepts, maximum and minimum points, and inflection points.
  - Then, make the curve pass through these points

Example

Example

Sketch the graph of

$$\frac{x^2}{\sqrt{x+1}}.$$

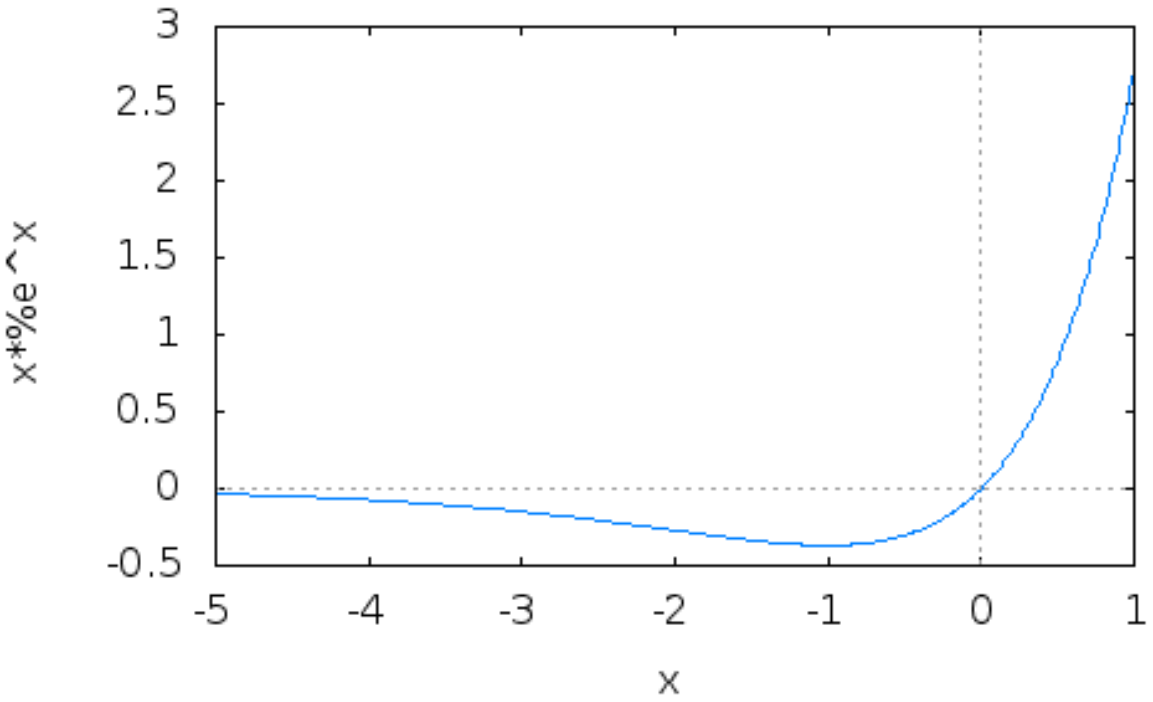


Example

Example

Sketch the graph of

$$f(x) = xe^x.$$

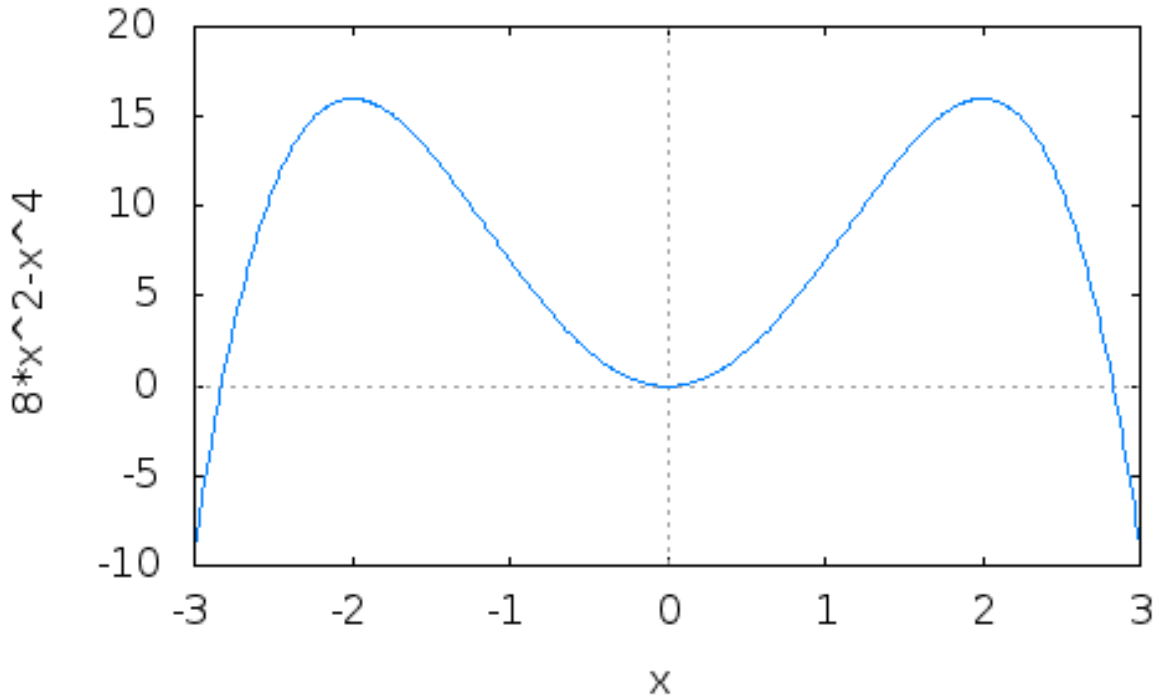


Example

Example

Sketch the graph of the curve

$$f(x) = 8x^2 - x^4.$$

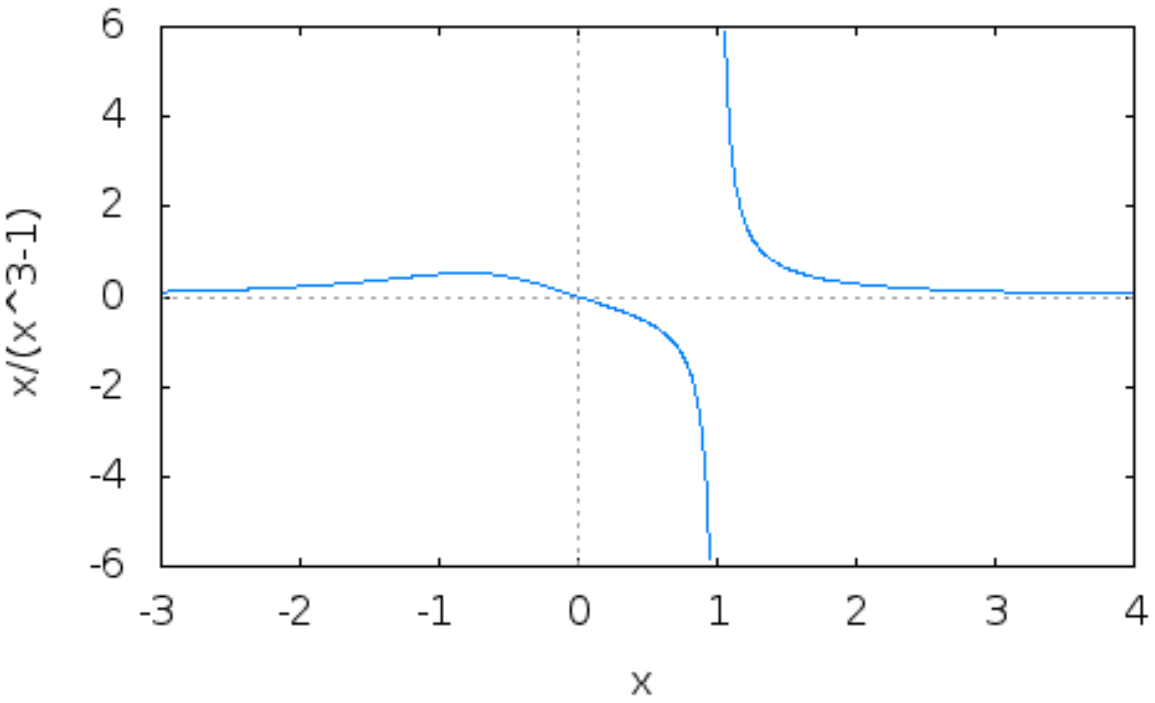


Example

Example

Sketch the graph of the curve

$$f(x) = \frac{x}{x^3 - 1}$$

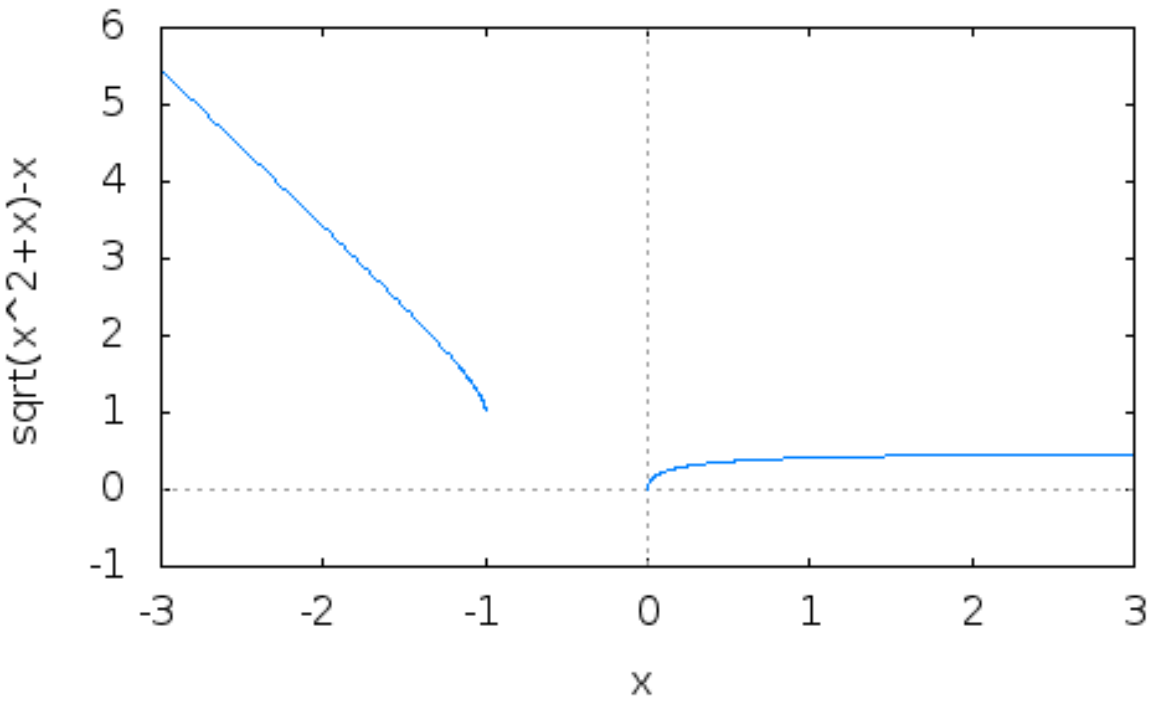


Example

Example

Sketch the graph of the curve

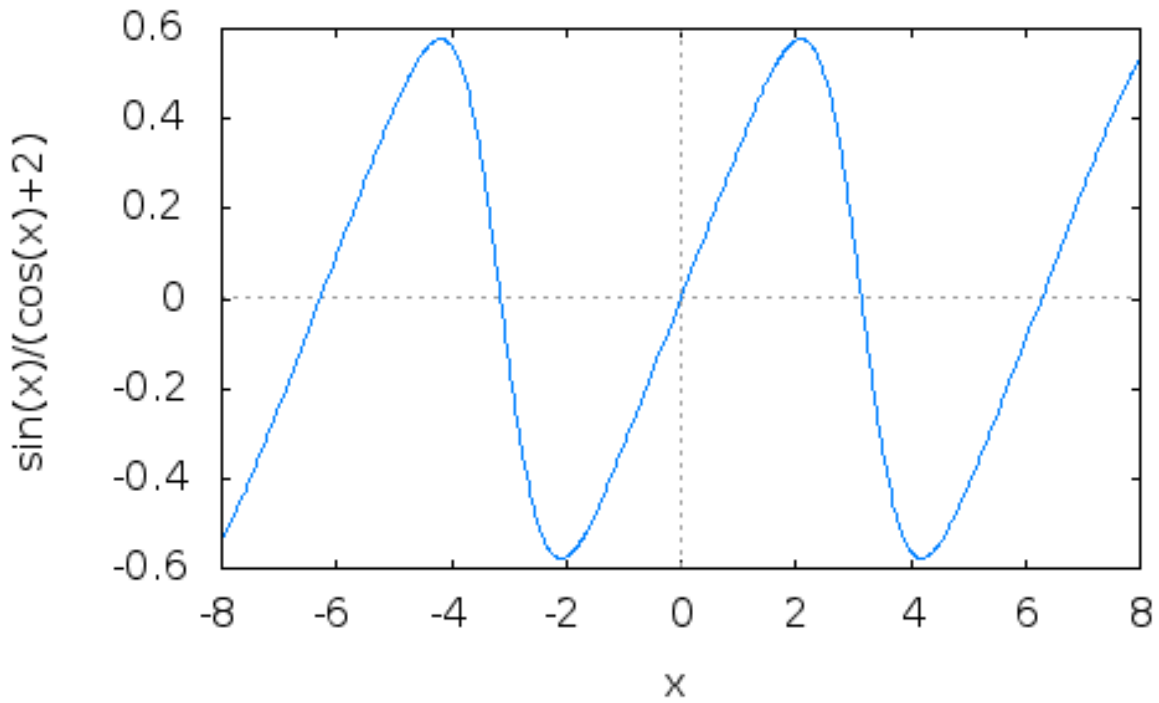
$$f(x) = \sqrt{x^2 + x} - x.$$



Example

Sketch the graph of the curve

$$f(x) = \frac{\sin x}{2 + \cos x}.$$

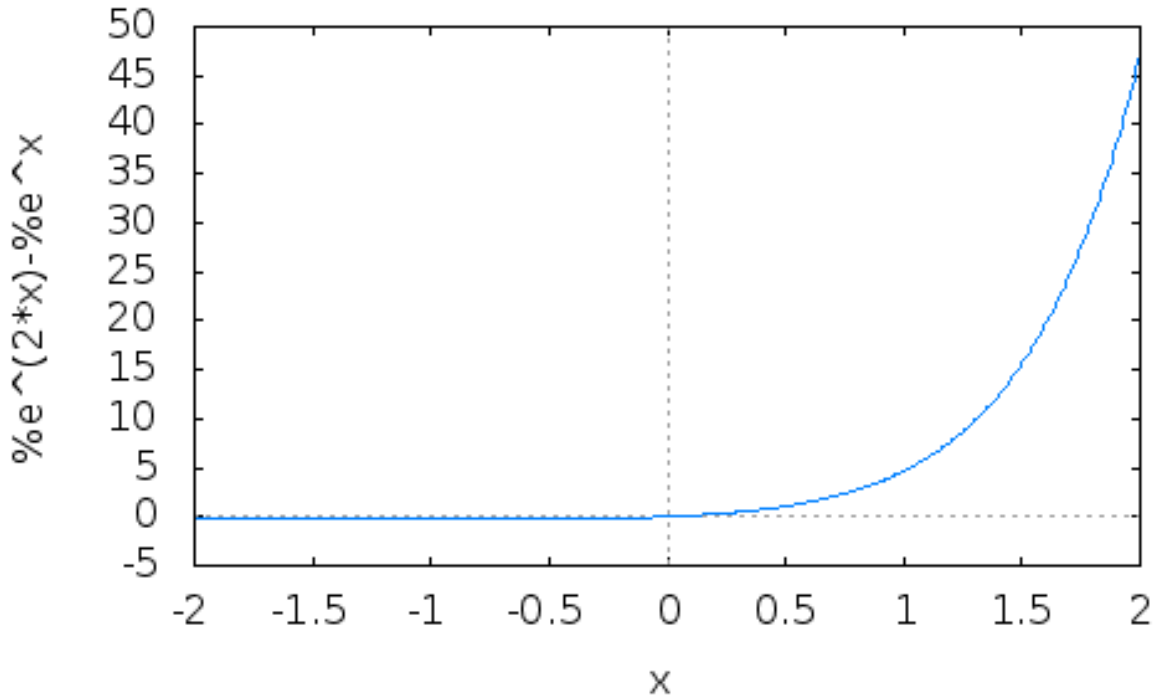


Example

Example

Sketch the graph of the curve

$$f(x) = e^{2x} - e^{x}.$$



Example

Example

Sketch the graph of the curve:

$$f(x) = \frac{\ln x}{x^2}.$$