

4.9 Antiderivatives

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Definition

A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I .

Fact

If F is an antiderivative of f on an interval I , then the most general antiderivative of f on I is

$$F(x) + C,$$

where C is an arbitrary constant.

Example

Find the most general antiderivatives of the following functions

- $f(x) = \cos x$
- $f(x) = \frac{1}{x}$
- $f(x) = x^n$

Table of Antidifferentiation formulas

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec^2 x$	$\tan x$
$x^n \ (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$1/x$	$\ln x $	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x	$\frac{1}{1+x^2}$	$\tan^{-1} x$
$\cos x$	$\sin x$		

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Example

Find the most general antiderivative of the function

- $f(x) = x(2 - x)^2$
- $f(x) = \sqrt[4]{x^3} + \sqrt[3]{x^4}$
- $f(x) = 3e^x + 5 \sec^2 x$
- $f(x) = \frac{2+x^2}{1+x^2}$

Example

Find f if

- $f''(x) = 6x + \sin x$
- $f'(x) = 2x - \frac{3}{x^4}$, $x > 0$ and $f(1) = 10$.
- $f''(x) = 4 - 6x - 40x^3$, $f(0) = 2$, $f'(0) = 1$.