

5.3 The Fundamental Theorem of Calculus

12/01/2010

Fact

Part I-antiderivative Suppose that f is a continuous function on the interval I containing the point a . Define the function F on I by the integral formula

$$F(x) = \int_a^x f(t)dt.$$

Then F is differentiable on I and $F'(x) = f(x)$. That is, F is an antiderivative of f on I .

Example

Example

- $\frac{d}{dx} \int_1^x t^2 dt$

Example

- $\frac{d}{dx} \int_1^x t^2 dt$
- $\frac{d}{dx} \int_1^{x^2} t^3 dt$

Example

- $\frac{d}{dx} \int_1^x t^2 dt$

- $\frac{d}{dx} \int_1^{x^2} t^3 dt$

- $\frac{d}{dx} \int_{x^2}^{x^3} e^{-t^2} dt$

Fact

Part II-evaluation: If $G(X)$ is any antiderivative of f on I (that is, $G'(x) = f(x)$ on I), then for any a and b in I ,

$$\int_a^b f(x)dx = G(b) - G(a).$$

Example

Example

- $\int_0^1 (x + 1) dx$

Example

- $\int_0^1 (x + 1) dx$
- $\int_0^{\pi/4} \sin x dx$

Example

- $\int_0^1 (x + 1) dx$
- $\int_0^{\pi/4} \sin x dx$
- $\int_0^{\pi/4} \sec^2 x$

Example

- $\int_0^1 (x + 1) dx$
- $\int_0^{\pi/4} \sin x dx$
- $\int_0^{\pi/4} \sec^2 x$
- $\int_0^3 e^x dx$

Example

- $\int_0^1 (x + 1) dx$
- $\int_0^{\pi/4} \sin x dx$
- $\int_0^{\pi/4} \sec^2 x$
- $\int_0^3 e^x dx$
- $\int_0^1 \frac{4}{t^2+1} dt$

5.4 Indefinite Integrals

Definition

The indefinite integral $\int f(x)dx$ of f is an antiderivative of f :

$$\int f(x)dx = F(x) \text{ means } F'(x) = f(x).$$

Table of indefinite integrals

$$\int u^r du = \frac{u^{r+1}}{r+1} + C, r \neq -1$$

$$\int \frac{1}{u} du = \ln |u| + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int e^u du = e^u + C$$

Example

Example

Find the general indefinite integral:

Example

Find the general indefinite integral:

- $\int (10x^4 + 2 \sec^2 x) dx$

Example

Example

Find the general indefinite integral:

- $\int(10x^4 + 2 \sec^2 x)dx$
- $\int(\csc^2 t - 2e^t)dt$

Example

Example

Find the general indefinite integral:

- $\int (10x^4 + 2 \sec^2 x) dx$
- $\int (\csc^2 t - 2e^t) dt$
- $\int \frac{\sin(2x)}{\sin(x)} dx$

Example

Evaluate the integral

Example

Evaluate the integral

- $\int_1^3 (1 + 2x - 4x^3) dx$

Example

Evaluate the integral

- $\int_1^3 (1 + 2x - 4x^3) dx$
- $\int_1^2 \frac{y+5y^7}{y^3} dy$

Example

Evaluate the integral

- $\int_1^3 (1 + 2x - 4x^3) dx$
- $\int_1^2 \frac{y+5y^7}{y^3} dy$
- $\int_{\pi/4}^{\pi/3} \sec x \tan x dx$