5.3 The Fundamental Theorem of Calculus

12/01/2010

Fact

Part I-antiderivative *Suppose that f is a continuous function on the interval I containing the point a. Define the function F on I by the integral formula*

$$F(x) = \int_a^x f(t)dt.$$

Then F is differentiable on I and $F'(x) = f(x)$. That is, F is an antiderivative of f on I.

Examples

Example

- $\frac{d}{dx} \int_1^x t^2 dt$
- $\frac{d}{dx} \int_1^{x^2} t^3 dt$
- $\frac{d}{dx} \int_{x^2}^{x^3} e^{-t^2} dt$

Fundamental Theorem of Calculus

Fact

Part II-evaluation: *If $G(X)$ is any antiderivative of f on I (that is, $G'(x) = f(x)$ on I), then for any a and b in I,*

$$\int_a^b f(x)dx = G(b) - G(a).$$

Examples

Example

- $\int_0^1 (x+1)dx$
- $\int_0^{\pi/4} \sin x dx$
- $\int_0^{\pi/4} \sec^2 x$
- $\int_0^3 e^x dx$
- $\int_0^1 \frac{4}{t^2+1} dt$

5.4 Indefinite Integrals

Definition

The indefinite integral $\int f(x)dx$ of f is an antiderivative of f:

$$\int f(x)dx = F(x) \text{ means } F'(x) = f(x).$$

$$
\begin{aligned}
\int u^r \, du &= \frac{u^{r+1}}{r+1} + C, r \neq -1 \\
\int \frac{1}{u} \, du &= \ln|u| + C \\
\int \sin u \, du &- -\cos u + C \\
\int \cos u \, du &= \sin u + C \\
\int \sec^2 u \, du &= \tan u - C \\
\int \sec u \tan u \, du &= \sec u + C \\
\int e^u \, du &= e^u + C
\end{aligned}
$$

Example

Example

Find the general indefinite integral:

- $\int (10x^4 + 2\sec^2 x)dx$
- $\int (csc^2 t - 2e^t)dt$
- $\int \frac{\sin(2x)}{\sin(x)}dx$

Examples

Example

Evaluate the integral

- $\int_1^3 (1 + 2x - 4x^3) dx$
- $\int_1^2 \frac{y+5y^7}{y^3} dy$
- $\int_{\pi/4}^{\pi/3} \sec x \tan x dx$