5.5 The Substitution Rule

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Differentials

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- If $y = x^3$, then $dy = 3x^2dx$.
- If $y = \sin 4x$, then $dy = 4\cos 4xdx$.

Reversing the Chain Rule

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- Integrating the right hand side reverses the chain rule and we get

$$\int f'(g(x))g'(x)dx = f(g(x)) + C.$$

- Substitute $u = g(x)$ and the differential $du = g'(x)dx$. When we make these two substitutions we get

$$\int f'(u)du = f(u) + C.$$

Examples

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- $\int \sin 2x dx$
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- $\int \frac{\sin x}{1+\cos^2 x}\,dx$
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- $\int \frac{\sin x}{1+\cos^2 x} dx$
- $\int \frac{e^x}{1+e^x} dx$
- $\int x^3 \sqrt{x^2 + 1} dx$

The substitution rule for definite integrals

Fact

If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$ then

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

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- $\int_{e}^{e^2} \frac{\ln x}{x} dx$

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- $\int_e^{e^2} \frac{\ln x}{x} dx$
- $\int_0^{\pi/4} \tan x dx$

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- $\int_0^1 x e^{-x^2} dx$
- $\int_0^{\pi/2} \cos x \sin(\sin x) dx$
- $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} dx$