5.5 The Substitution Rule

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12/06/2010

Differentials

- Differentials are a very useful technique for solving integrals.
- If $y = x^3$, then $dy = 3x^2 dx$.
- If $y = \sin 4x$, then $dy = 4 \cos 4x dx$.

Reversing the Chain Rule

- If $u = g(x)$ is a function of x, and f is a function of u, then the chain rule tells us that

$$(f(g(x)))' = f'(g(x))g'(x).$$

- Integrating the right hand side reverses the chain rule and we get

$$\int f'(g(x))g'(x)dx = f(g(x)) + C.$$

- Substitute $u = g(x)$ and the differential $du = g'(x)dx$. When we make these two substitutions we get

$$\int f'(u)du = f(u) + C.$$

Examples

Example

- $\int e^{7x} dx$
- $\int \sin 2x dx$
- $\int \tan x dx$
- $\int \frac{x}{x^2+1} dx$
- $\int \sqrt{x} \sin(1 + x^{3/2}) dx$
- $\int \frac{x^2+1}{x^3+3x+2} dx$
- $\int \frac{\ln x}{x} dx$
- $\int \frac{\sin(\ln x)}{x} dx$
- $\int \frac{\sin x}{1+\cos^2 x} dx$
- $\int \frac{e^x}{1+e^x} dx$
- $\int x^3 \sqrt{x^2 + 1} dx$

The substitution rule for definite integrals

Fact

If g' *is continuous on* $[a, b]$ *and* f *is continuous on the range of* $u = g(x)$ *then*

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

Examples

Example

- $\int_e^{e^2} \frac{\ln x}{x} dx$
- $\int_0^{\pi/4} \tan x dx$
- $\int_0^1 x e^{-x^2} dx$
- $\int_0^{\pi/2} \cos x \sin(\sin x) dx$
- $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} dx$