# 6.1 Area between curves

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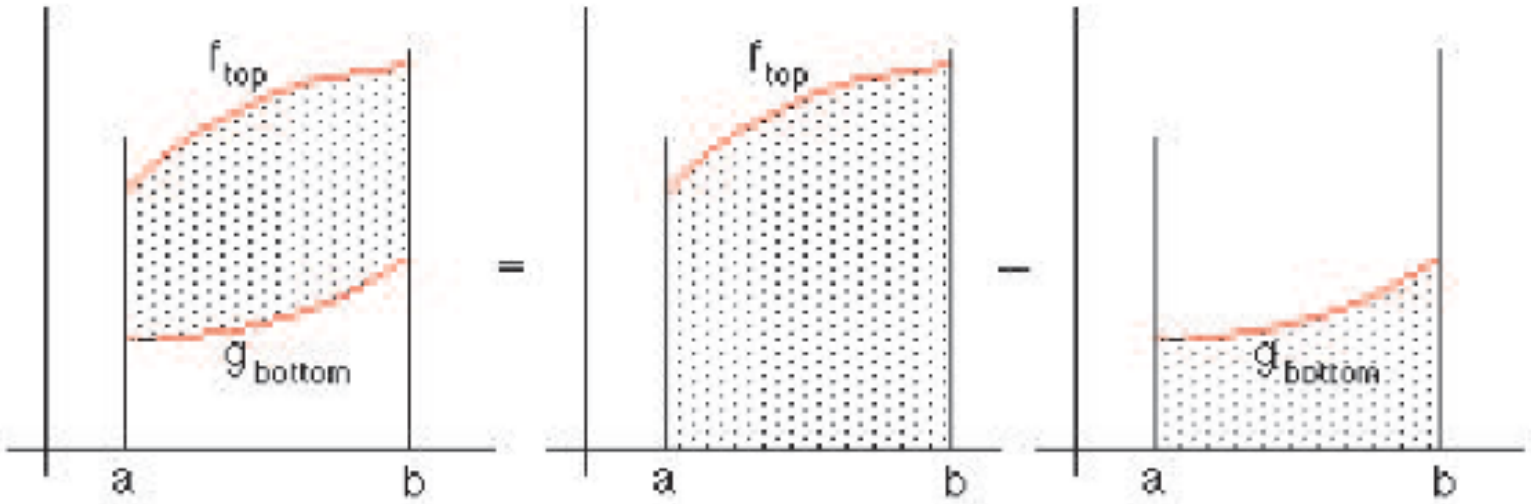
- Suppose we are given two continuous functions, $f_{top}$ and $g_{bottom}$ defined on the interval $[a, b]$, with $g_{bottom}(x) \leq f_{top}(x)$ for all $x$ in the interval.
- How do we find the area bounded by the two functions over that interval?

# Area between curves



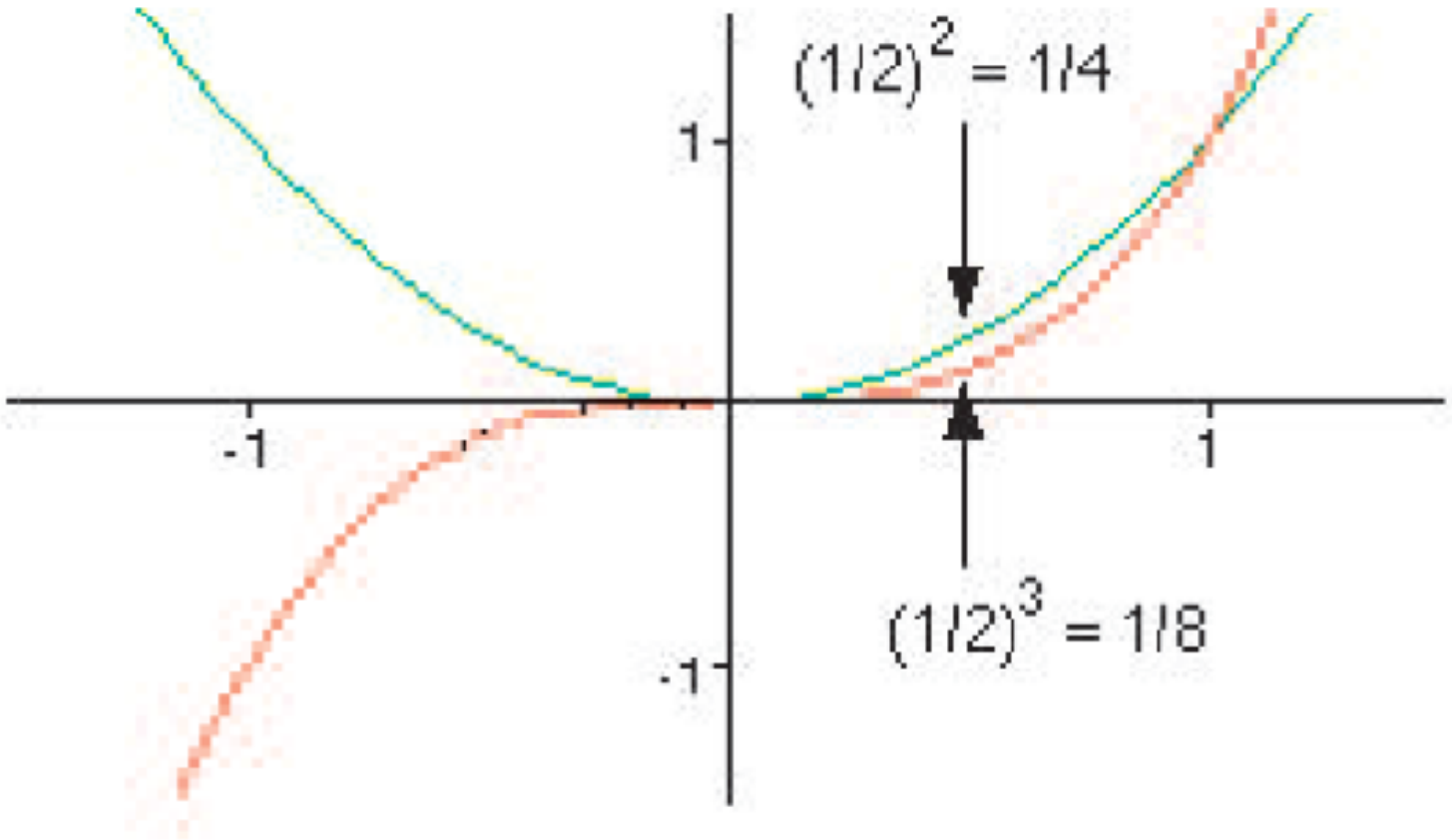
# The Area Between Two Curves

$$\int_a^b f_{top}(x)\,dx - \int_a^b g_{bottom}(x)\,dx = \int_a^b \left(f_{top}(x) - g_{bottom}(x)\right)\,dx$$

# Example

## Example

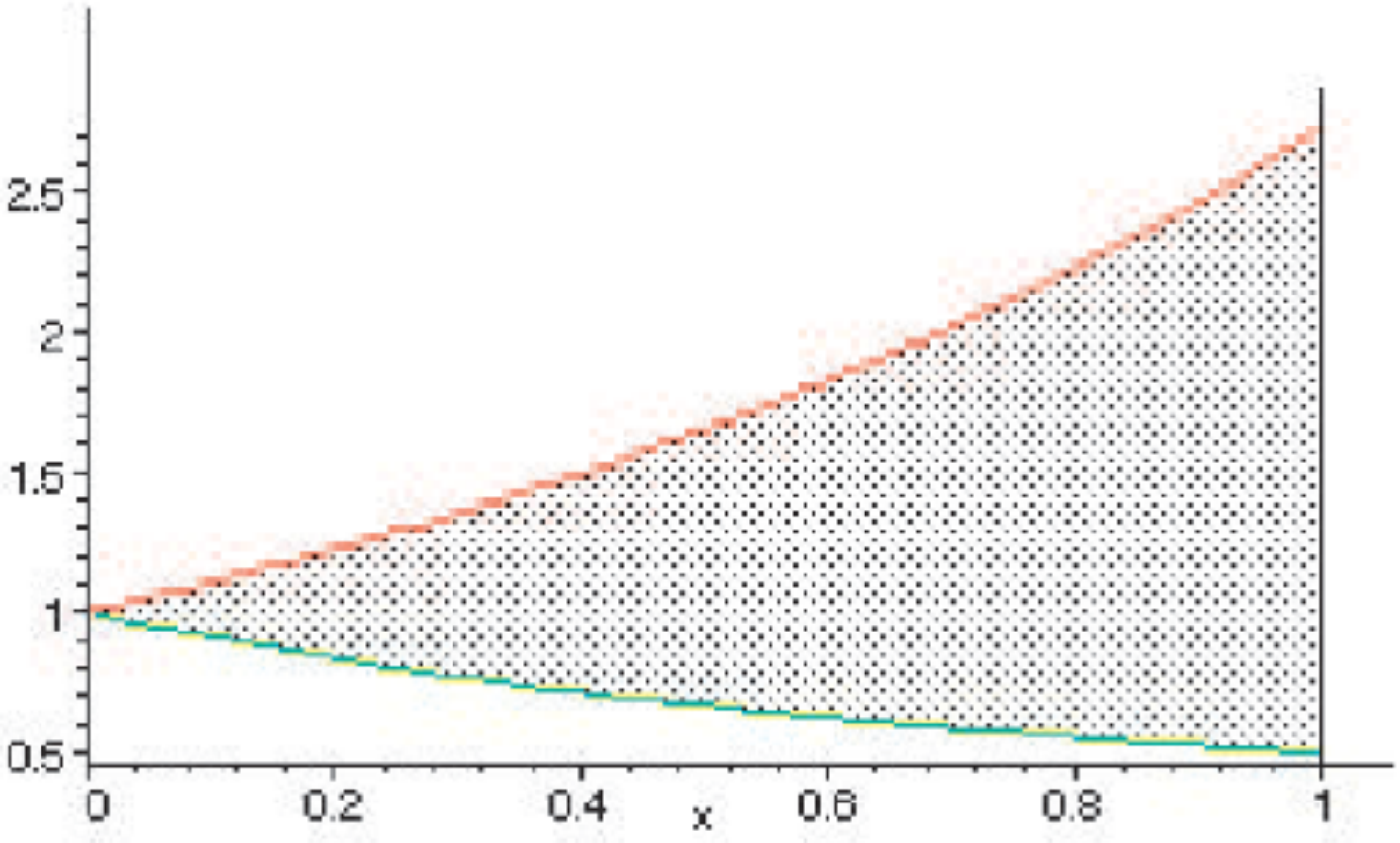
Find the area of the region between the graphs of $y = x^2$ and $y = x^3$ for $0 \leq x \leq 1$.



# Example

## Example

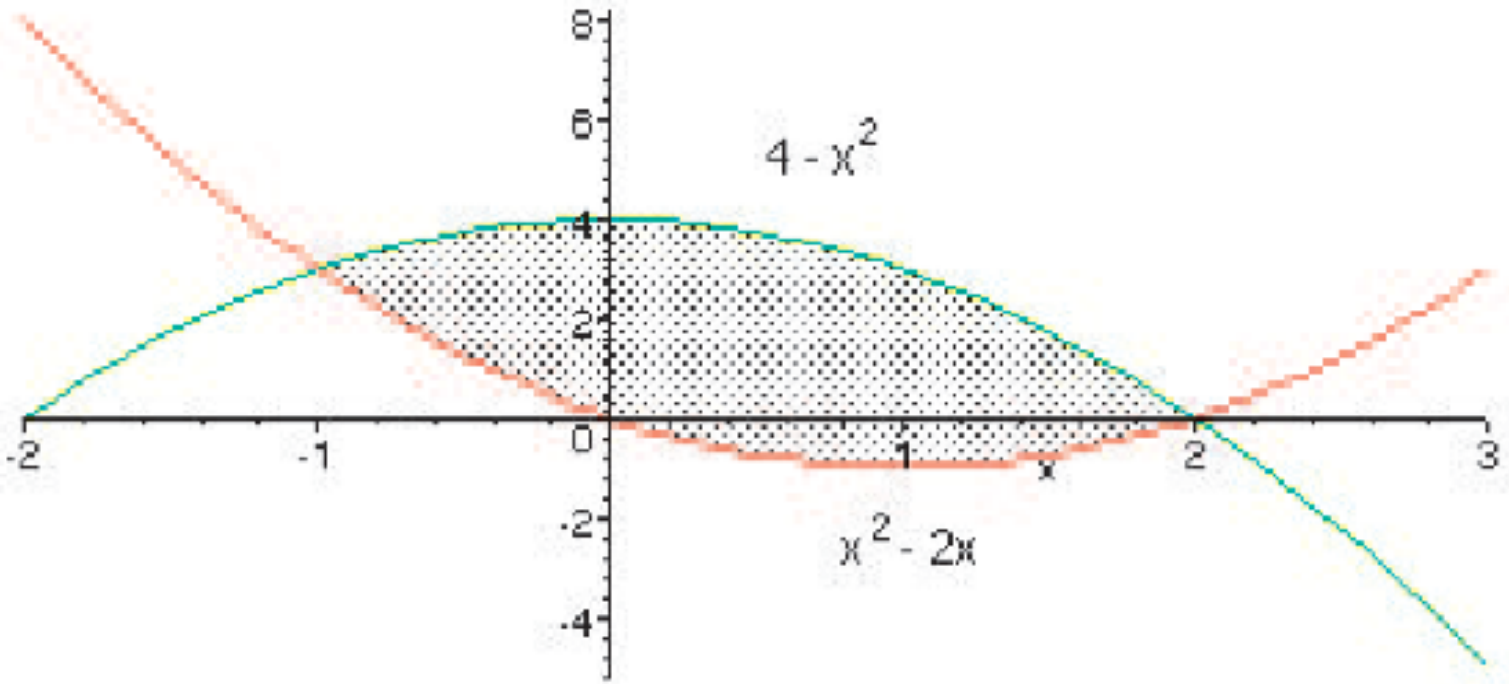
Find the area of the region between $y = e^x$ and $y = 1/(1+x)$ on the interval $[0, 1]$.



# Example

## Example

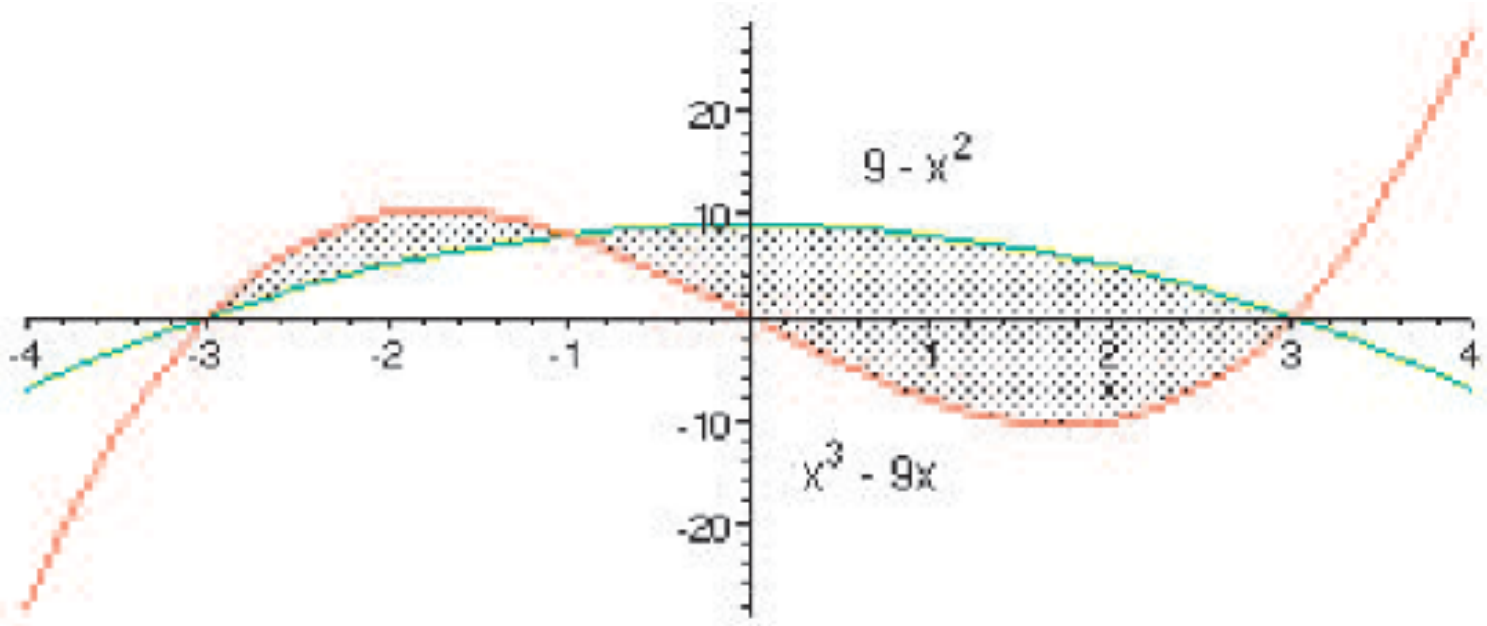
Find the area of the region bounded by $y = x^2 - 2x$ and $y = 4 - x^2$.



# Example

## Example

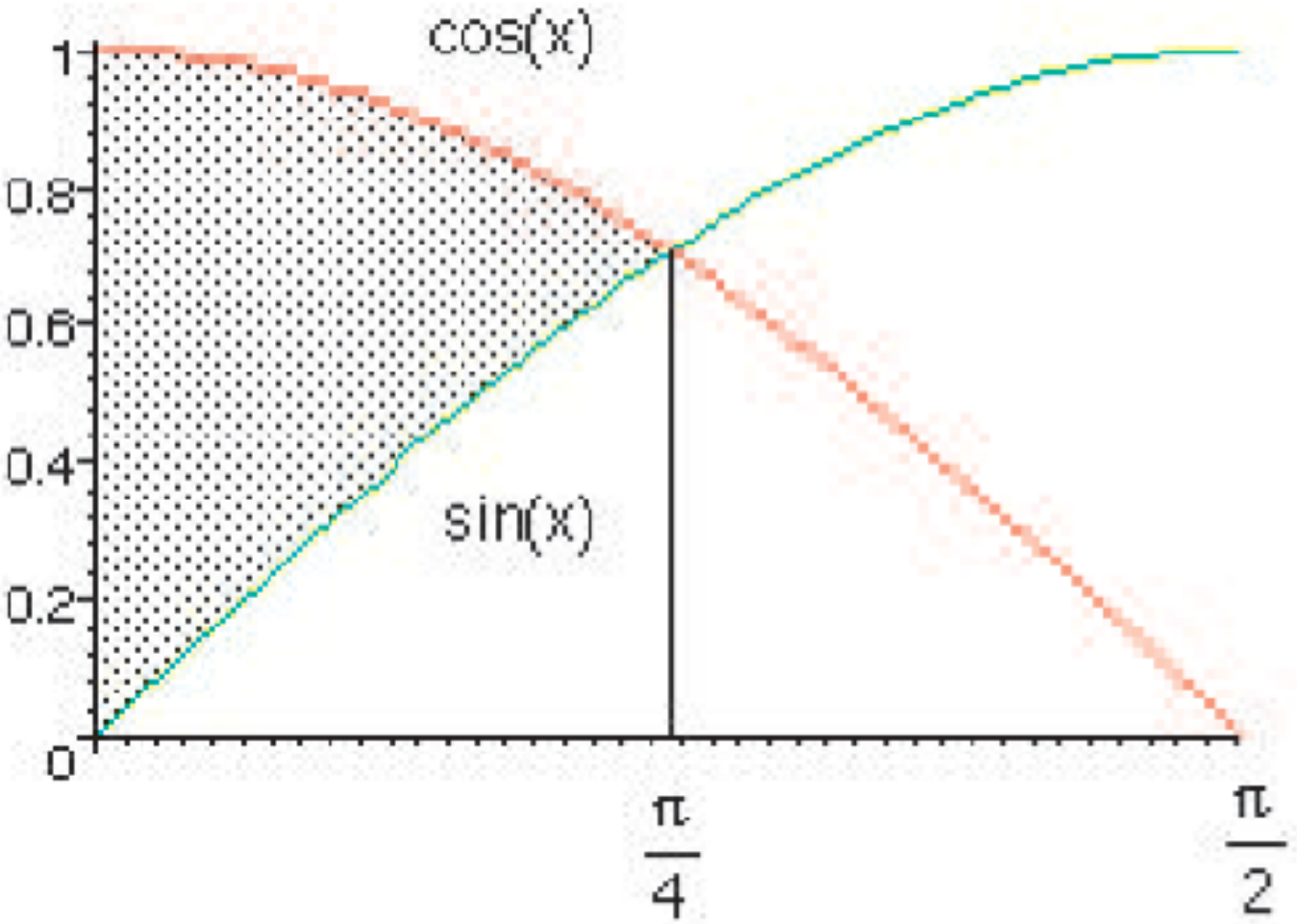
Find the area of the region bounded by the two curves $y = x^3 - 9x$ and $y = 9 - x^2$.



# Example

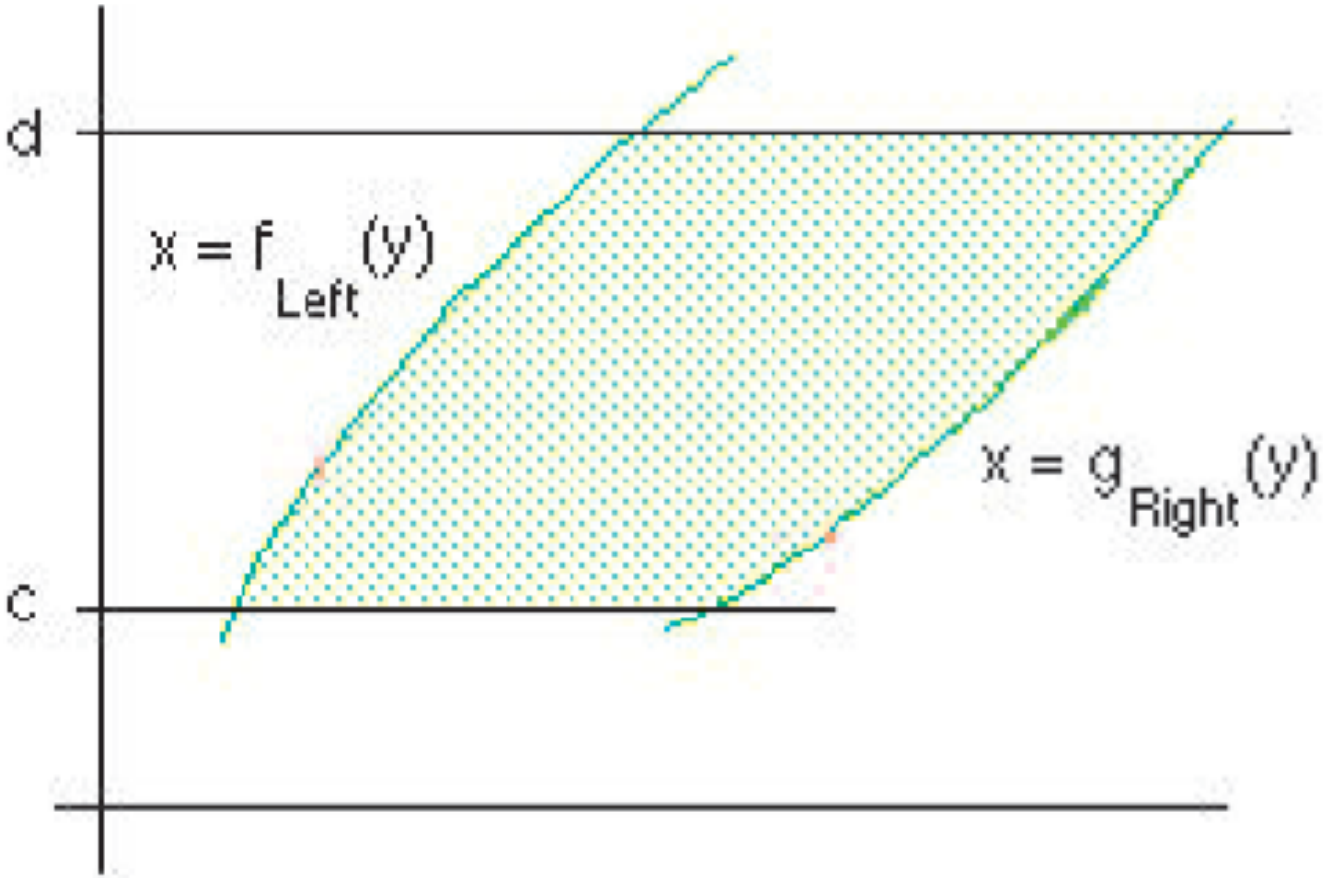
## Example

Find the area between $\sin x$ and $\cos x$ on $[0, \pi/4]$.



# Functions of $y$

- We could just as well consider two functions of $y$, say, $x = f_{Left}(y)$ and $x = g_{Right}(y)$ defined on the interval $[c, d]$.



# Example

## Example

Find the area under the graph of $y = \ln x$ and above the interval $[1, e]$ on the $x$-axis.