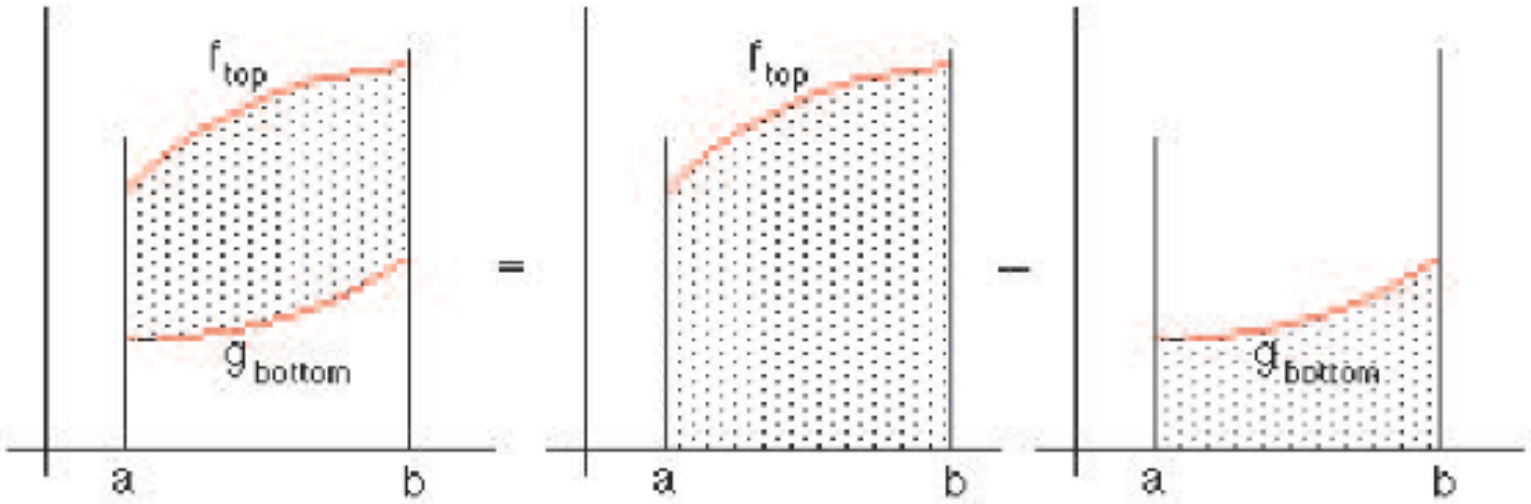
6.1 Area between curves

12/09/2010

Areas Between Curves

- We know that if f is a continuous nonnegative function on the interval $[a, b]$, then $\int_a^b f(x)dx$ is the area under the graph of f and above the interval.
- Suppose we are given two continuous functions, f_{top} and g_{bottom} defined on the interval $[a, b]$, with $g_{bottom}(x) \leq f_{top}(x)$ for all x in the interval.
- How do we find the area bounded by the two functions over that interval?

Area between curves



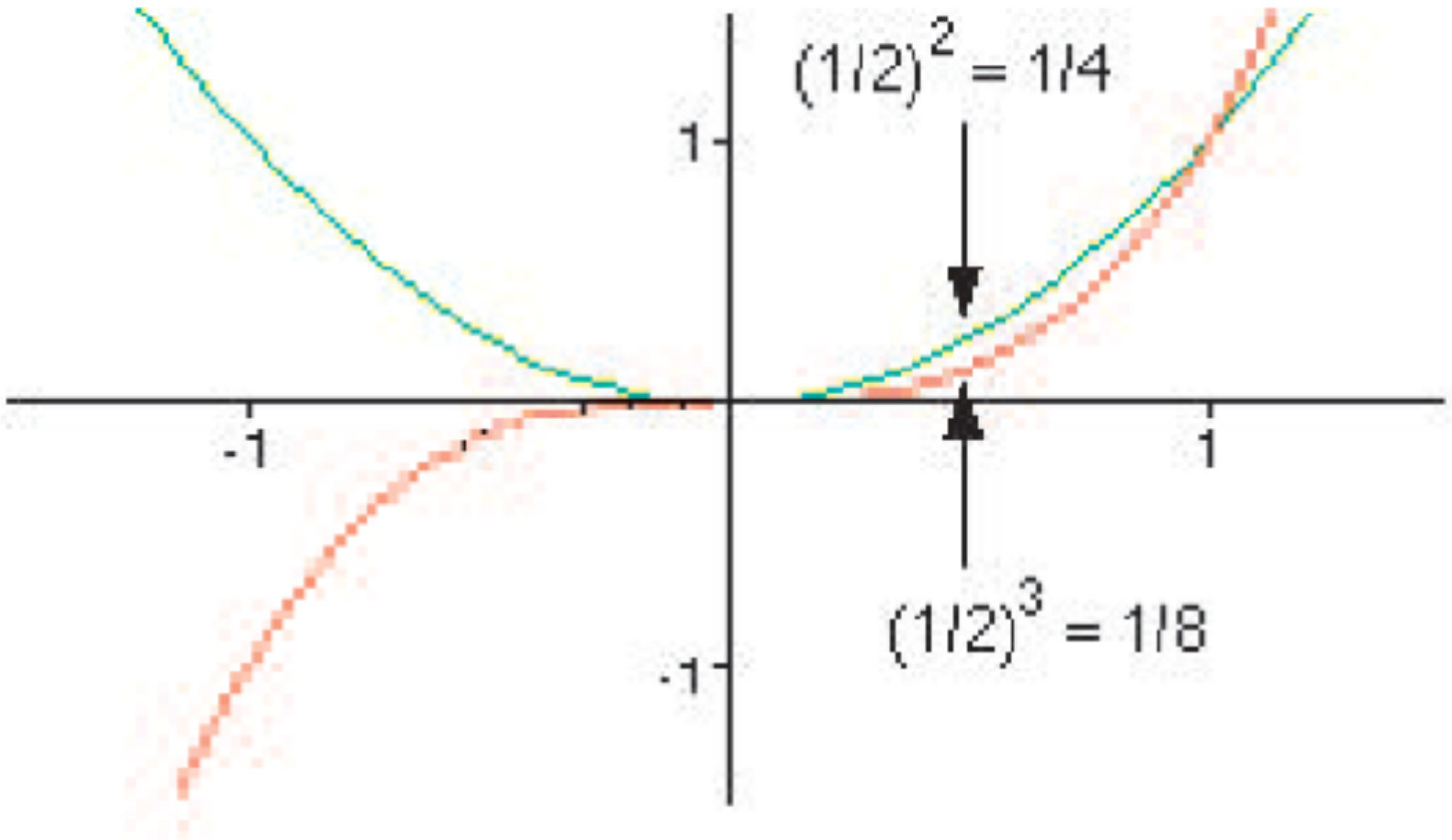
The Area Between Two Curves

$$\int_a^b f_{top}(x)\,dx - \int_a^b g_{bottom}(x)\,dx = \int_a^b \left(f_{top}(x) - g_{bottom}(x)\right)\,dx$$

Example

Example

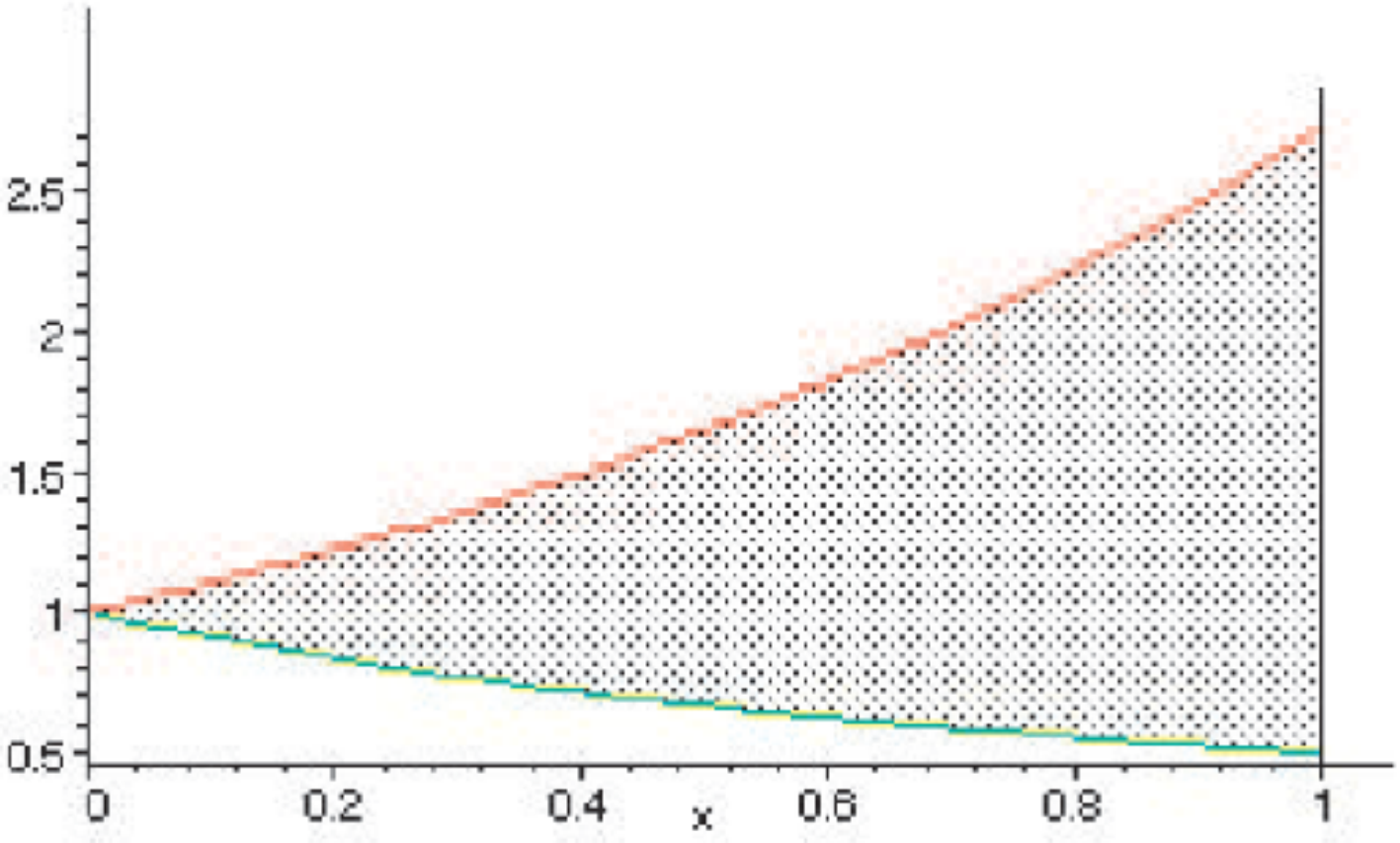
Find the area of the region between the graphs of $y = x^2$ and $y = x^3$ for $0 \leq x \leq 1$.



Example

Example

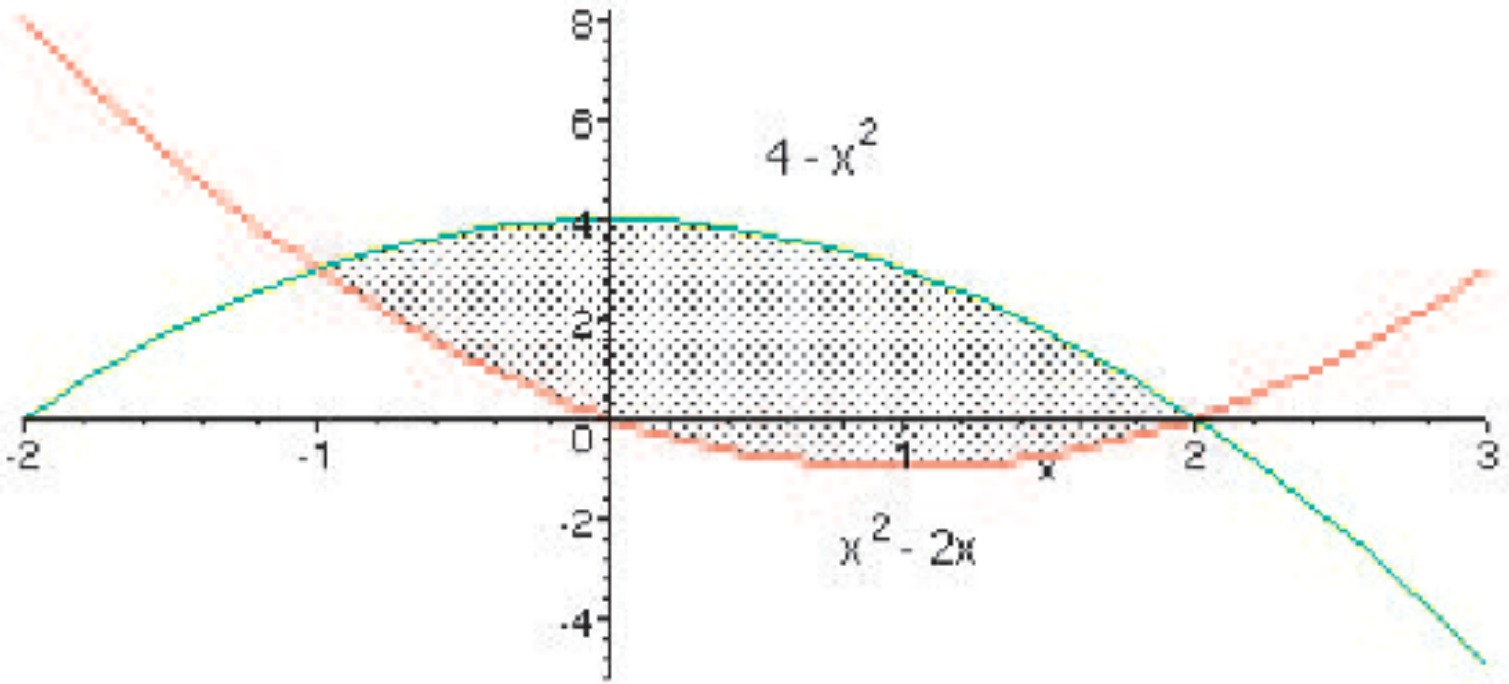
Find the area of the region between $y = e^x$ and $y = 1/(1+x)$ on the interval $[0, 1]$.



Example

Example

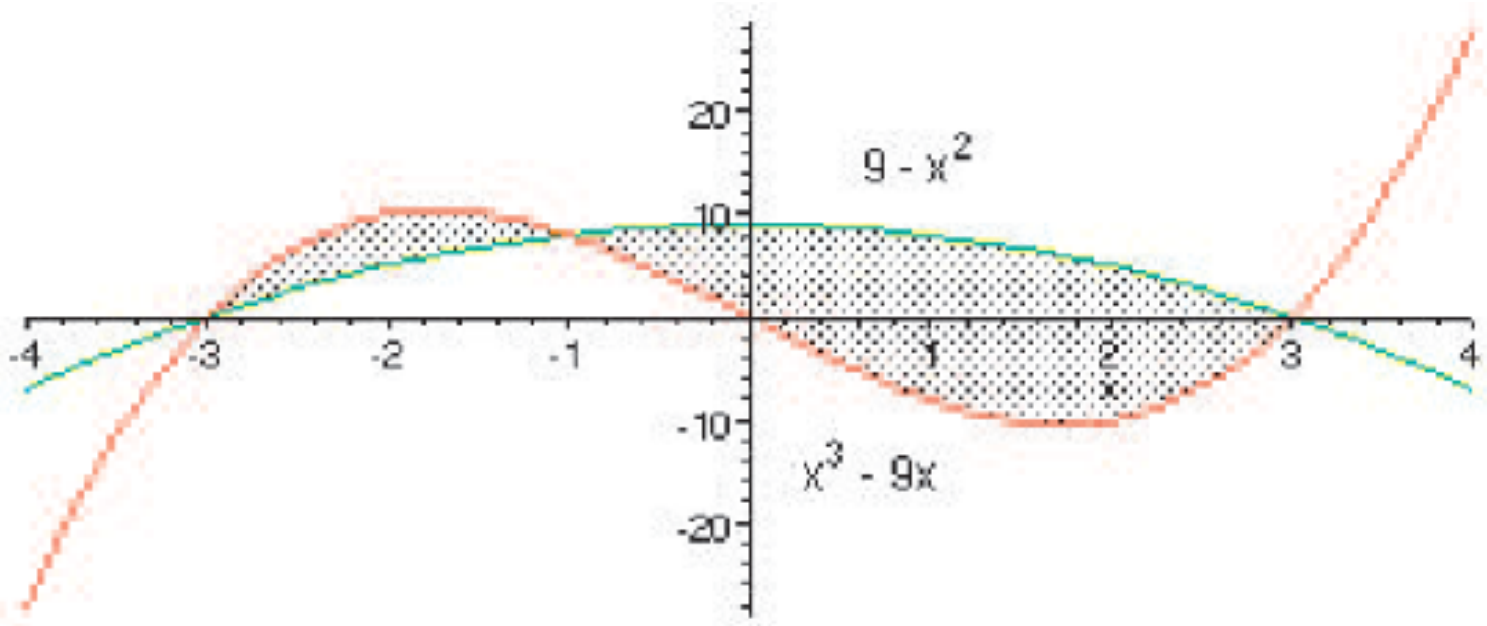
Find the area of the region bounded by $y = x^2 - 2x$ and $y = 4 - x^2$.



Example

Example

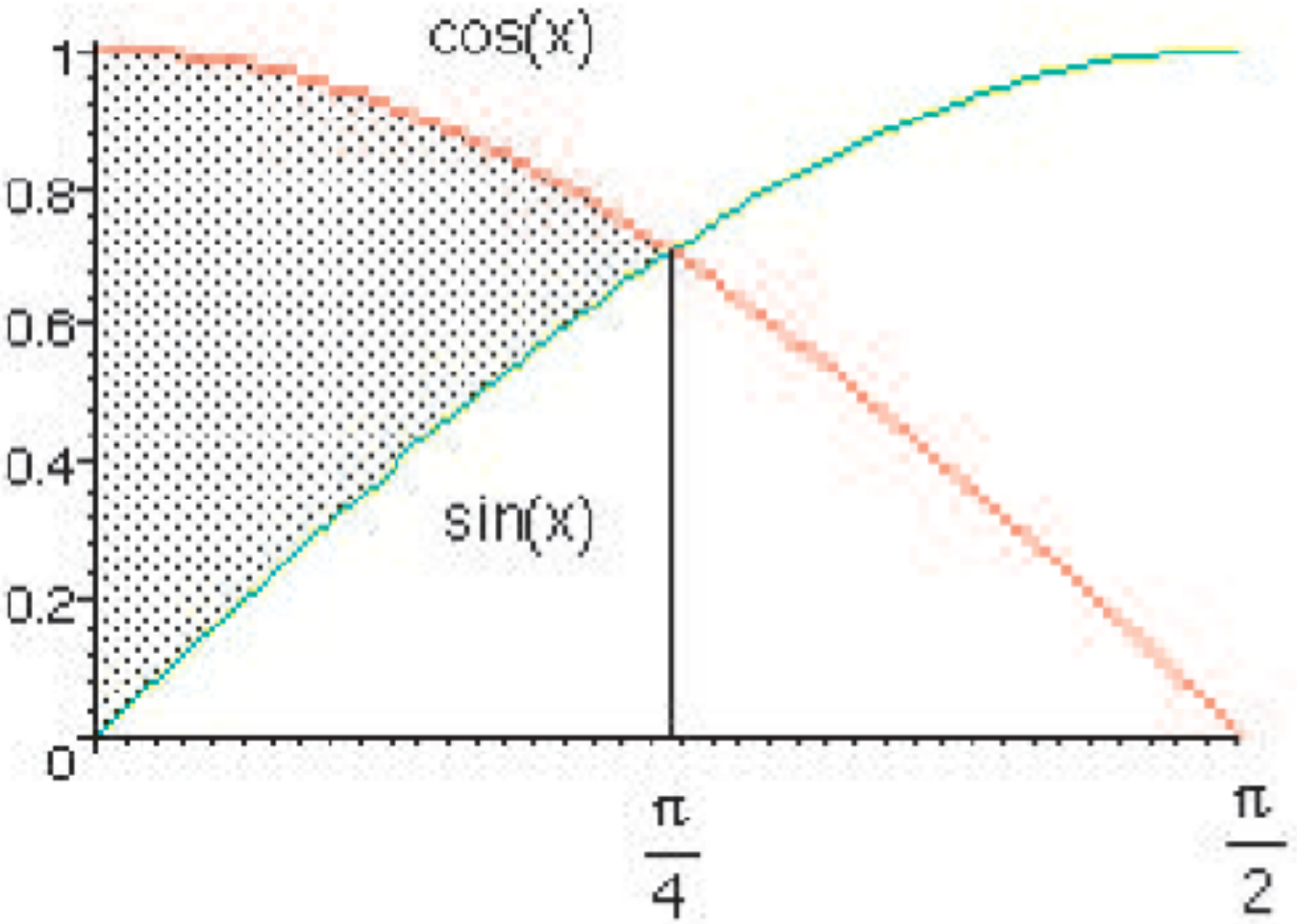
Find the area of the region bounded by the two curves $y = x^3 - 9x$ and $y = 9 - x^2$.



Example

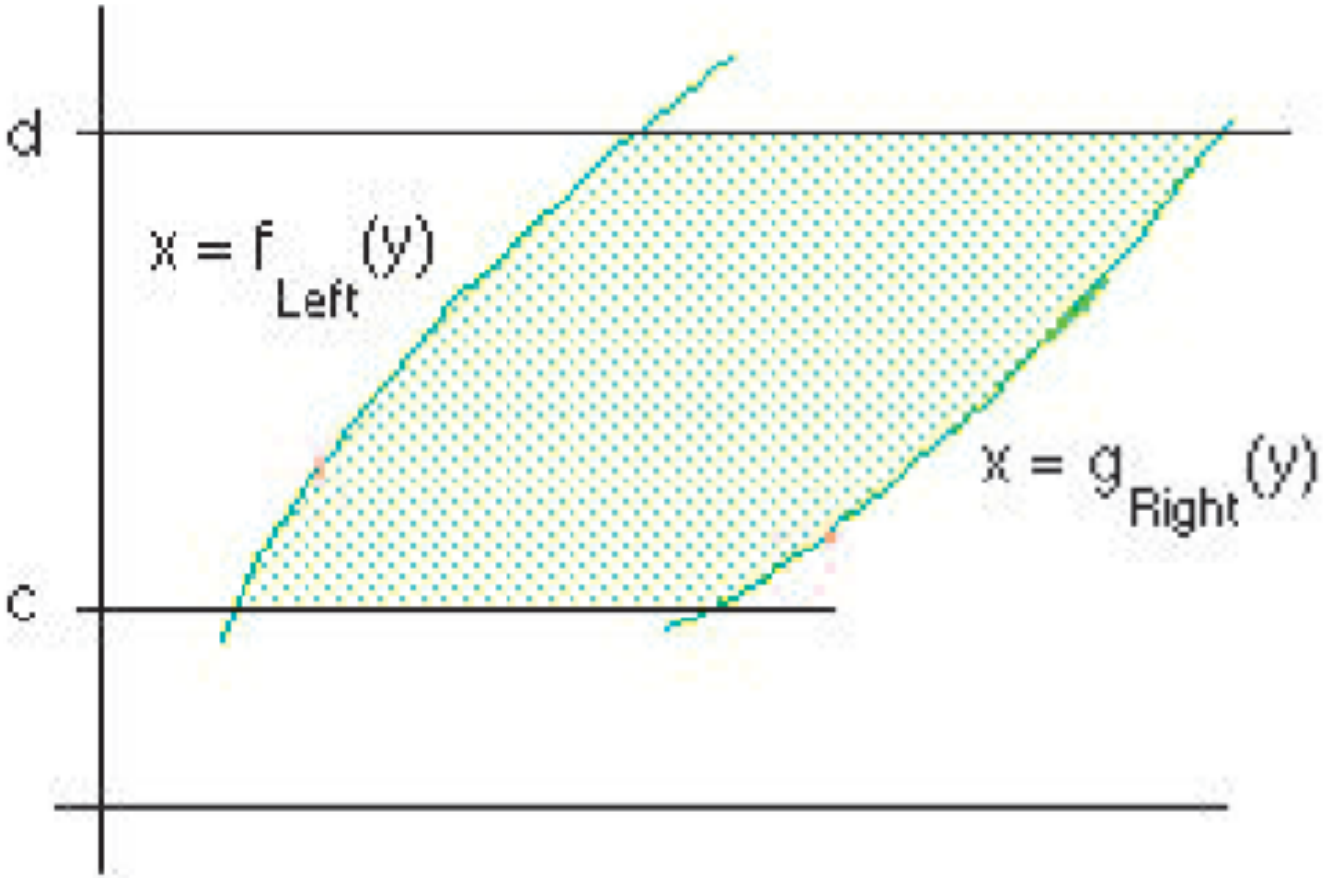
Example

Find the area between $\sin x$ and $\cos x$ on $[0, \pi/4]$.



Functions of y

- We could just as well consider two functions of y, say, $x = f_{Left}(y)$ and $x = g_{Right}(y)$ defined on the interval $[c, d]$.



Example

Example

Find the area under the graph of $y = \ln x$ and above the interval $[1, e]$ on the x-axis.