2.2 The Limit of a Function

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The Legacy of Galileo, Newton, and Leibniz

- Galileo
  - was interested in falling bodies.
  - forged a new scientific methodology - *observe nature, construct experiments to test what you observe, and construct theories that explain the observations*.
- Newton
  - was able, using his new tools of calculus, to explain why falling bodies behave in this way: an object, falling under the influence of gravity, will have constant acceleration of $9.8m/sec^2$.
  - his laws of motion and of universal gravitation drew under one simple mathematical theory Newton's laws of falling bodies, Kepler's laws of planetary motion, the motion of a simple pendulum, and virtually every other instance of dynamic motion observed in the universe.
- Leibniz
  - co-inventor of calculus, took a slightly different point of view but also studied rates of change in a general setting.

Newton's Question

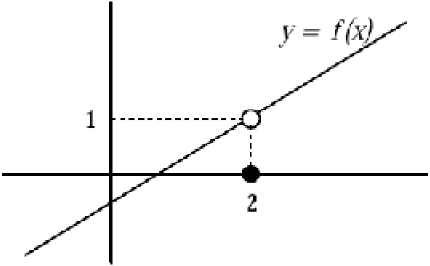
- How do we find the velocity of a moving object at time t?
- What in fact do we mean by *velocity* of the object at the instant of time t?? We know how to find the average velocity of an object during a time interval $[t_1, t_2]$?

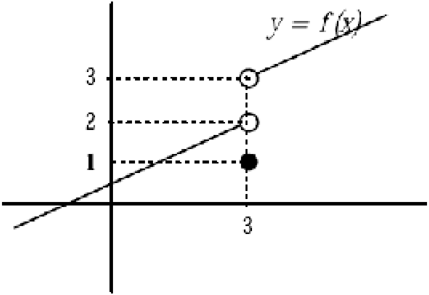
Limit of a Function

Definition

We say that a function f approaches the limit L as x approaches a, written $\lim_{x \to a} f(x) = L$, if we can make $f(x)$ as close to L as we please by taking x sufficiently close to a.

Example




y = f(x)
3
2
1
3

One-Sided Limits

Definition

We write

$$\lim_{x \to a^-} f(x) = L$$

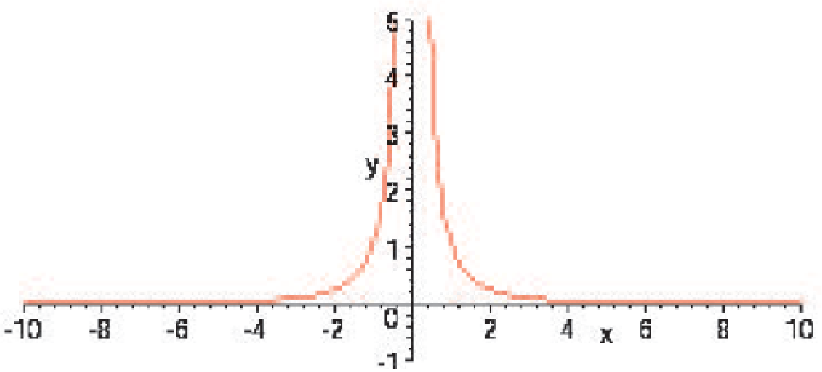
and say the left-hand limit of $f(x)$ as x approaches a –or the limit of f(x) as x approaches a from the left– is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a and x less than a.

Fact

The limit of f as $x \to a$ exists if and only if both the right-hand and left-hand limits exist and have the same value. i.e.

$$\lim_{x \to a} f(x) = L \Leftrightarrow \lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = L.$$

Compute the limit $\lim_{x \to 0} 1/x^2$.



Definition

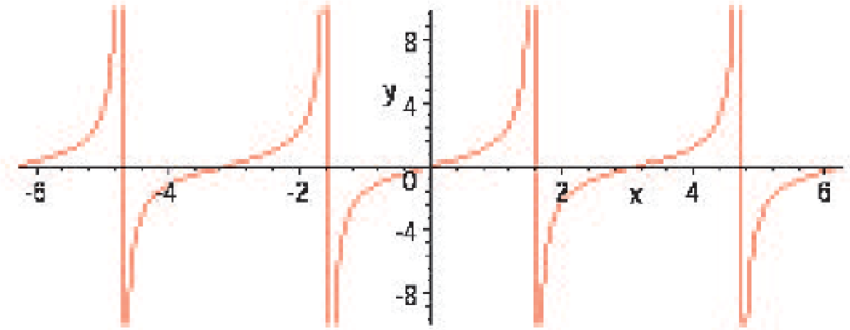
Let f be a function defined on both sides of a, except possibly at a itself. Then,

$$\lim_{x \to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large—as large as we please—by taking x sufficiently close to a, but not equal to a.

Example

Evaluate $\lim_{x \to \pi/2} \tan x$.



Definition

The line $x = a$ is called a vertical asymptote of the curve $y = f(x)$ if at least one of the following statement is true:

$$\lim_{x\to a} f(x) = \infty \quad \lim_{x\to a^+} f(x) = \infty \quad \lim_{x\to a^-} f(x) = \infty$$
$$\lim_{x\to a} f(x) = -\infty \quad \lim_{x\to a^+} f(x) = -\infty \quad \lim_{x\to a^-} f(x) = -\infty$$