2.3 Limit Laws

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Fact (Limit Laws)

If $\lim_{x\to a} f(x) = A$ *and* $\lim g(x) = B$ *both exist, then*

1. $\lim_{x\to a}(f(x) + g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = A + B$
2. $\lim_{x\to a}(f(x) - g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = A - B$
3. $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x) = cA.$
4. $\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = A \cdot B$
5. $\lim_{x\to a}(f(x)/g(x)) = \lim_{x\to a} f(x)/\lim_{x\to a} g(x) = A/B$ $(B \neq 0)$.

Example

If $\lim_{x\to 2} f(x) = 2$ and $\lim_{x\to 2} g(x) = 3$ find

$$\lim_{x\to 2} \frac{f(x) + 3g(x)}{f(x)g(x)}.$$

Direct Substitution Property

Fact (Direct Substitution Property)

If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x \to a} f(x) = f(a).$$

Examples

Computing limits

Compute the following limits:

- $\lim_{x \to 2} \frac{x-2}{x+3}$
- $\lim_{x \to 1} \frac{x^2-1}{x-1}$

Examples

Compute the following limits:

1. $\lim_{x\to 1} \frac{x^2-2x+3}{x^3+3x-1}$
2. $\lim_{x\to 0} \frac{|x|}{x}$
3. Let $f(x) = x^2$. Compute

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Properties of Limits

Theorem

If $f(x) \leq g(x)$ when x is near a (except possibly at a) and the limits of f and g both exist as x approaches a, then

$$\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$$

Theorem (Squeeze Theorem)

If

$$f(x) \leq g(x) \leq h(x)$$

when x is near (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L,$$

then

$$\lim_{x \to a} g(x) = L.$$

Example

Show that
$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.$$