2.3 Limit Laws (cont'd)

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Fact (Limit Laws)

Recall

If $\lim_{x\to a} f(x) = A$ *and* $\lim g(x) = B$ *both exist, then*

1. $\lim_{x\to a}(f(x) + g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = A + B$
2. $\lim_{x\to a}(f(x) - g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = A - B$
3. $\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x) = cA.$
4. $\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = A \cdot B$
5. $\lim_{x\to a}(f(x)/g(x)) = \lim_{x\to a} f(x)/\lim_{x\to a} g(x) = A/B\ (B \neq 0).$

Fact (Direct Substitution Property)

If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x\to a} f(x) = f(a).$$

Examples

Computing limits

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4. $\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$

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4. $\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$
5. $\lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4}$

Examples

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2. $\lim_{x \to 0} \frac{f(x)}{x}$.

Properties of Limits

Theorem

If $f(x) \leq g(x)$ when x is near a (except possibly at a) and the limits of f and g both exist as x approaches a, then

$$\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$$

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Theorem (Squeeze Theorem)

If

$$f(x) \leq g(x) \leq h(x)$$

when x is near (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L,$$

then

$$\lim_{x \to a} g(x) = L.$$

Examples

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$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.$$

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1. Show that
$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.$$
2. Prove that
$$\lim_{x \to 0^+} \sqrt{x}e^{\sin(\pi/x)} = 0.$$