2.3 Limit Laws (cont'd)

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Limit Laws

Fact (Limit Laws)

Recall

If $\lim_{x\to a} f(x) = A$ *and* $\lim g(x) = B$ *both exist, then*

1. $\lim_{x\to a}(f(x) + g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = A + B$
2. $\lim_{x\to a}(f(x) - g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = A - B$
3. $\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x) = cA.$
4. $\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = A \cdot B$
5. $\lim_{x\to a}(f(x)/g(x)) = \lim_{x\to a} f(x)/\lim_{x\to a} g(x) = A/B$ $(B \neq 0)$.

Fact (Direct Substitution Property)

If f *is a polynomial or a rational function and* a *is in the domain of* f*, then*

$$\lim_{x\to a} f(x) = f(a).$$

Examples

Computing limits

Compute the following limits:

1. $\lim_{t \to 0} \frac{\sqrt{t^2+16}-4}{t}$
2. $\lim_{x \to 4} \frac{x^2-4x}{x^2-3x-4}$
3. $\lim_{x \to 1} \frac{x^3-1}{x^2-1}$
4. $\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2+t} \right)$
5. $\lim_{x \to -4} \frac{\sqrt{x^2+9}-5}{x+4}$

Examples

If

$$\lim_{x \to 0} \frac{f(x)}{x^2} = 5,$$

find the following limits

1. $\lim_{x \to 0} f(x)$
2. $\lim_{x \to 0} \frac{f(x)}{x}$.

Properties of Limits

Theorem

If $f(x) \leq g(x)$ *when* x *is near* a *(except possibly at* a*) and the limits of* f *and* g *both exist as* x *approaches* a*, then*

$$\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$$

Theorem (Squeeze Theorem)

If

$$f(x) \leq g(x) \leq h(x)$$

when x *is near (except possibly at* a *) and*

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L,$$

then

$$\lim_{x \to a} g(x) = L.$$

Examples

Example

1. Show that
$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.$$
2. Prove that
$$\lim_{x \to 0^+} \sqrt{x} e^{\sin(\pi/x)} = 0.$$