2.5 Continuity

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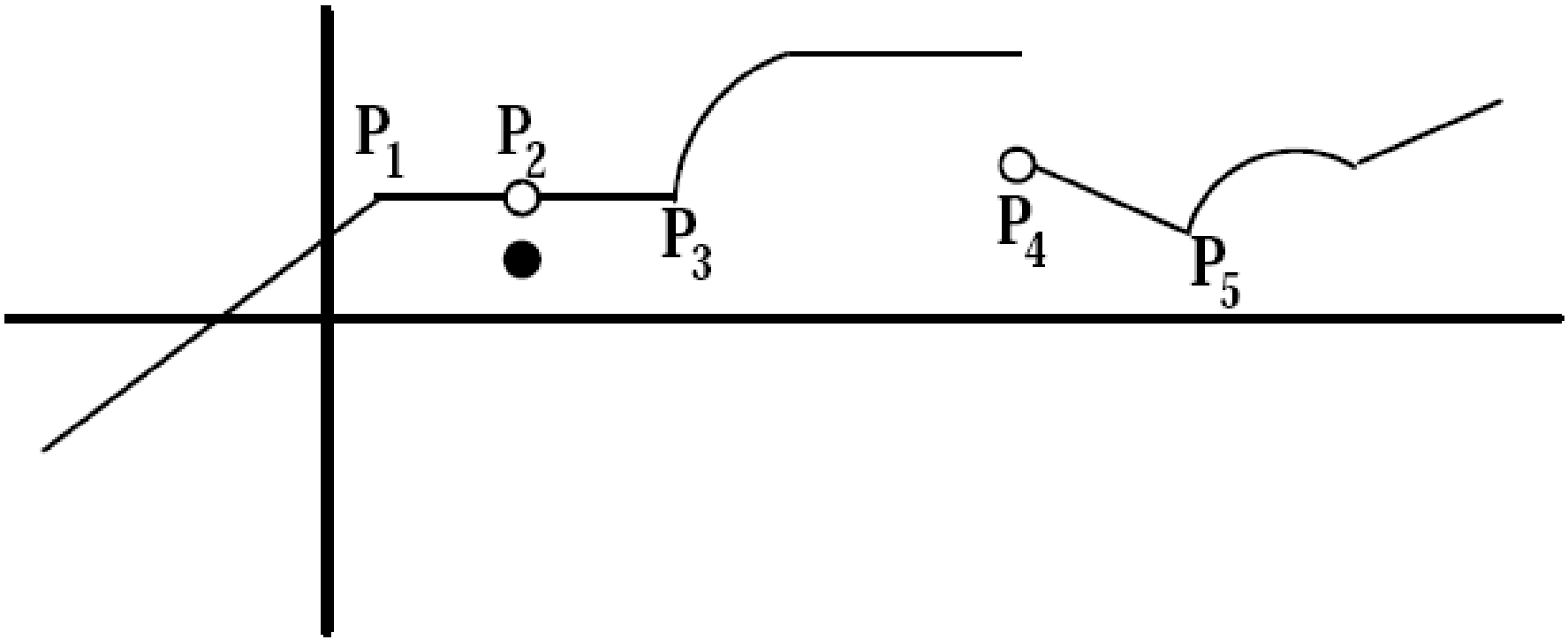
Example



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- If it is not continuous there, i.e. if either the limit does not exist or is not equal to $f(a)$ we will say that the function is discontinuous at a.

Note

1. The function f is defined at the point $x = a$,
2. $\lim_{x \to a} f(x)$ exists, call it L, and
3. $L = f(a)$.

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continuous at $x = 1$?

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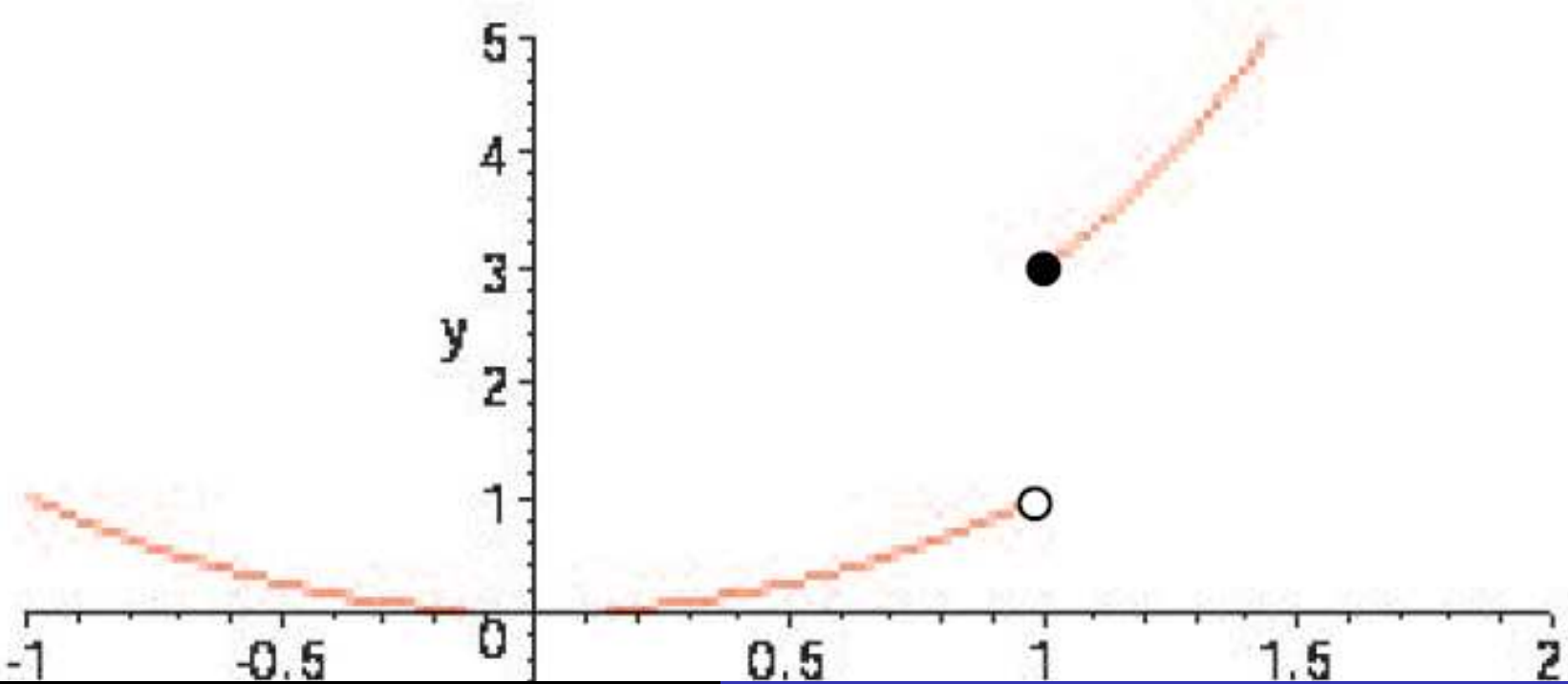

Right Continuity and Left Continuity

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- A function f is right continuous at a point c if it is defined on an interval $[c, d]$ lying to the right of c and if $\lim_{x \to c^+} f(x) = f(c)$.
- Similarly it is left continuous at c if it is defined on an interval $[d, c]$ lying to the left of c and if $\lim_{x \to c^-} f(x) = f(c)$.

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1. If $x = a$ is an interior point of the domain of f, then $\lim_{x \to a} f(x) = f(a)$.
2. If $x = a$ is not an interior point of the domain but is an endpoint of the domain, then f must be right or left continuous at $x = a$, as appropriate.

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- A function f is said to be a continuous function if it is continuous at every point of its domain.
- A point of discontinuity of a function f is a point in the domain of f at which the function is not continuous.

Facts

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Continuous Functions

- *All polynomials,*
- *Rational functions,*
- *Trigonometric functions,*
- *The absolute value function, and*
- *The exponential and logarithm functions*

are continuous.

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It has a *continuous extension*

$$F(x) = \begin{cases} f(x) & \text{if } x \text{ is in the domain of } f \\ 4 & \text{if } x = 2. \end{cases}$$

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We can "remove" the discontinuity by redefining the value of f at $\pi/3$.

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Suppose that $f(x)$ is defined piecewise as

$$f(x) = \begin{cases} -x^2 + 1 & \text{if } x < 2 \\ x + k & \text{if } x \geq 2 \end{cases}$$

Find a value of the constant k such that f is continuous at $x = 2$.