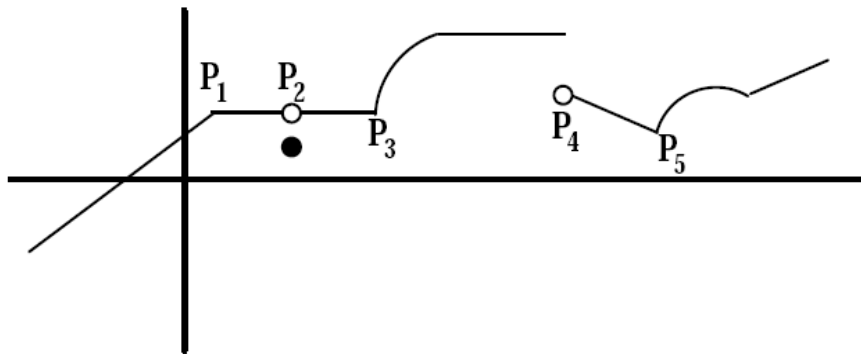


2.5 Continuity

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09/08/2010

Example



Definition

- A function is continuous at a point a of its domain if $\lim_{x \rightarrow a} f(x) = f(a)$.
- If it is not continuous there, i.e. if either the limit does not exist or is not equal to $f(a)$ we will say that the function is discontinuous at a .

- 1 The function f is defined at the point $x = a$,
- 2 $\lim_{x \rightarrow a} f(x)$ exists, call it L , and
- 3 $L = f(a)$.

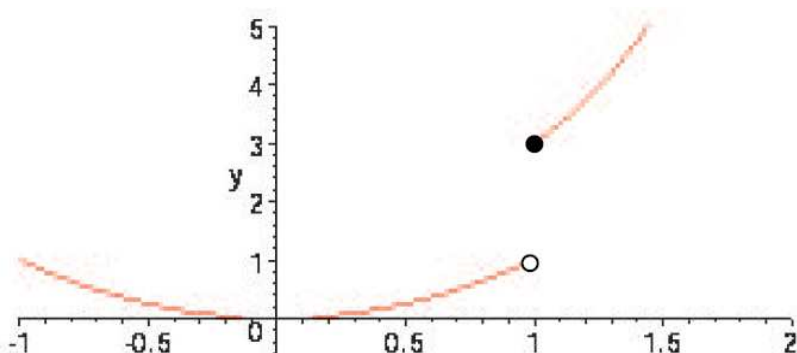
Example

Example

Is the function

$$f(x) = \begin{cases} x^2 & x < 1 \\ x^3 + 2 & 1 \leq x \end{cases}$$

continuous at $x = 1$?



Definition

- A function f is right continuous at a point c if it is defined on an interval $[c, d]$ lying to the right of c and if $\lim_{x \rightarrow c^+} f(x) = f(c)$.
- Similarly it is left continuous at c if it is defined on an interval $[d, c]$ lying to the left of c and if $\lim_{x \rightarrow c^-} f(x) = f(c)$.

Definition

A function f is continuous at a point $x = a$ if a is in the domain of f and:

- 1 If $x = a$ is an interior point of the domain of f , then $\lim_{x \rightarrow a} f(x) = f(a)$.
- 2 If $x = a$ is not an interior point of the domain but is an endpoint of the domain, then f must be right or left continuous at $x = a$, as appropriate.

Definition

- A function f is said to be a continuous function if it is continuous at every point of its domain.
- A point of discontinuity of a function f is a point in the domain of f at which the function is not continuous.

Fact

Continuous Functions

- *All polynomials,*
- *Rational functions,*
- *Trigonometric functions,*
- *The absolute value function, and*
- *The exponential and logarithm functions*

are continuous.

Example

- The rational function $f(x) = \frac{x^2-4}{x-2}$ is a continuous function.
- The domain is all real numbers except 2.
- $\lim_{x \rightarrow 2} f(x) = 4$ exists.

It has a *continuous extension*

$$F(x) = \begin{cases} f(x) & \text{if } x \text{ is in the domain of } f \\ 4 & \text{if } x = 2. \end{cases}$$

Example

The function

$$f(x) = \begin{cases} \sin x & x \neq \pi/3 \\ 0 & x = \pi/3 \end{cases}$$

is discontinuous at $\pi/3$.

We can “remove” the discontinuity by redefining the value of f at $\pi/3$.

Example

Suppose that $f(x)$ is defined piecewise as

$$f(x) = \begin{cases} -x^2 + 1 & \text{if } x < 2 \\ x + k & \text{if } x \geq 2 \end{cases}$$

Find a value of the constant k such that f is continuous at $x = 2$.