2.5 Continuity (cont'd)

Marius Ionescu

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Definition

- If c is a discontinuity of a function f, and if $\lim_{x \to c} f(x) = L$ exists, then c is called a removable discontinuity. The discontinuity is removed by defining $f(c) = L$.
- If f is not defined at c but $\lim_{x \to c} f(x) = L$ exists, then f has a continuous extension to $x = c$ by defining $f(c) = L$.

Example

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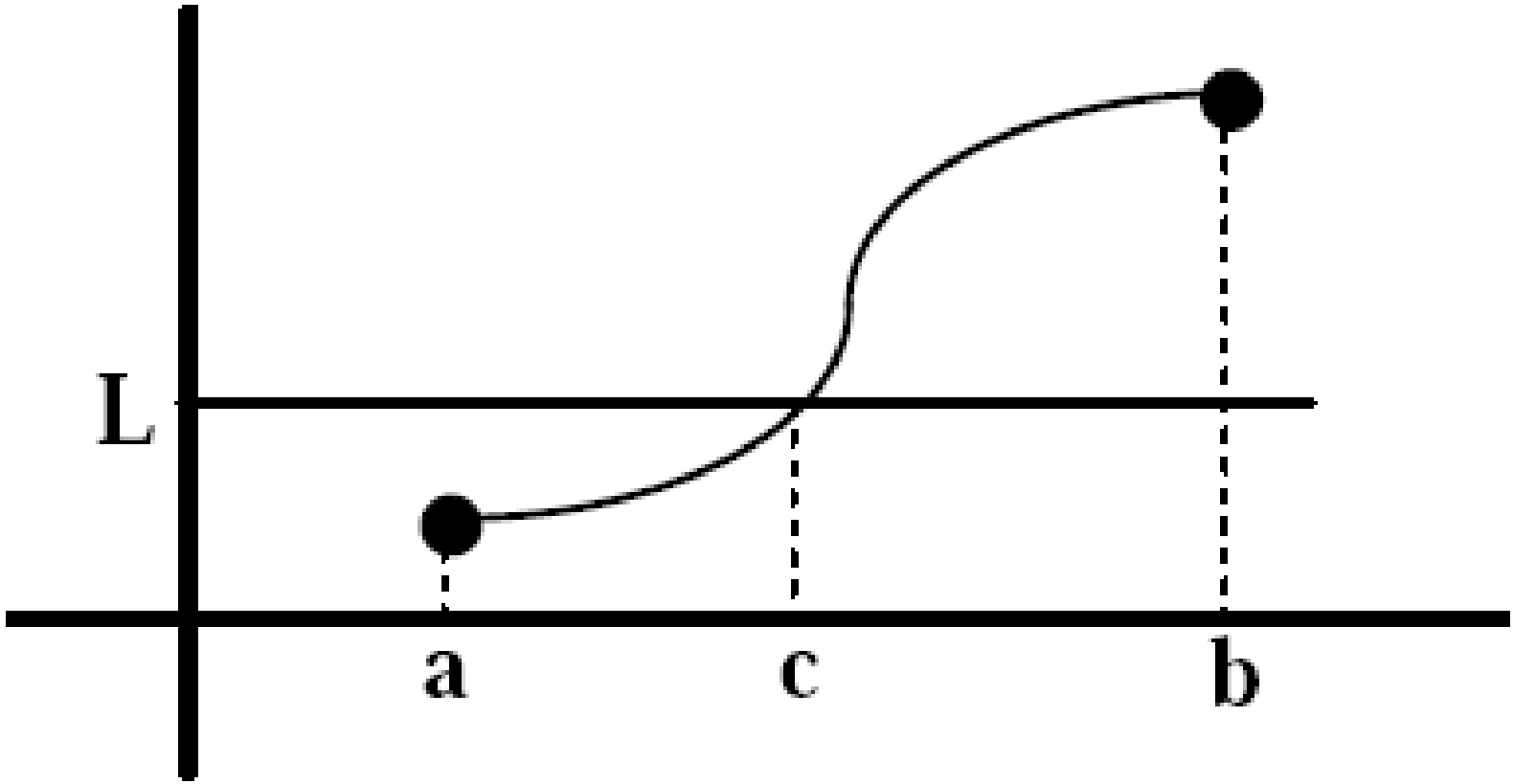
Does the following function have a removable discontinuity at $a = 1$?

$$f(x) = \frac{x^4 - 1}{x - 1}$$

Find the continuous extension of f to 1.

The Intermediate Value Theorem

If a function f is continuous on a closed interval $[a, b]$, and if $f(a) < L < f(b)$ (or $f(a) > L > f(b)$), then there exists a point c in the interval $[a, b]$ such that $f(c) = L$.



Examples

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1. Show that the equation $x^5 - 3x + 1 = 0$ has a solution in the interval $[0, 1]$.
2. Does the equation $1/x = 0$ have a solution?

More examples

Examples

- Show that f is continuous on $(-\infty, \infty)$

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4. \end{cases}$$

- Find the numbers at which f is discontinuous

$$f(x) = \begin{cases} x + 1 & \text{if } x \leq 1 \\ \frac{1}{x} & \text{if } 1 < x < 3 \\ \sqrt{x - 3} & \text{if } x \geq 3. \end{cases}$$