2.6 Limits at Infinity: Horizontal asymptotes

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Definition

Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that the value of $f(x)$ approaches L as the value of x approaches $+\infty$. This means that $f(x)$ can be made as close to L as we please by taking the value of x sufficiently large.
Similarly,

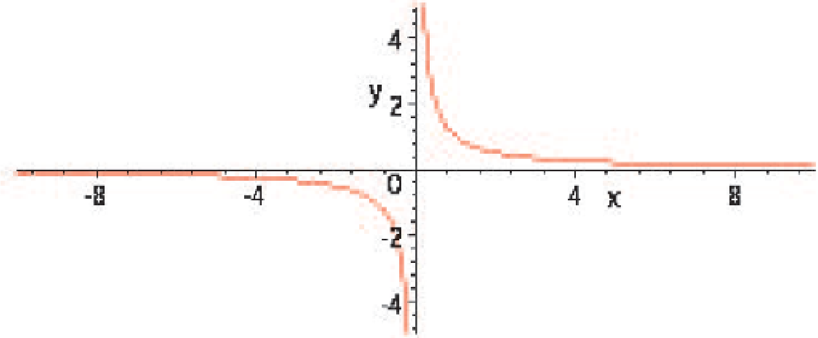
$$\lim_{x \to -\infty} f(x) = L$$

means that $f(x)$ can be made as close to L as we please by taking the value of x sufficiently small (in the negative direction).

Example

Example

$\lim_{x\to\infty} 1/x = 0.$



More Examples

Examples

Evaluate the limits:

1. $\lim_{x\to\infty} \frac{x-1}{x^3+2}$
2. $\lim_{x\to\infty} \frac{3x^2-2x+1}{4x^2-1}$
3. $\lim_{x\to\infty} \frac{x}{\sqrt{3x^2+2}}$
4. $\lim_{x\to\infty} \left(\sqrt{x^2+4}-x\right)$.

Horizontal Asymptotes

Definition

The line $y = L$ is called a horizontal asymptote of the curve $y = f(x)$ if either

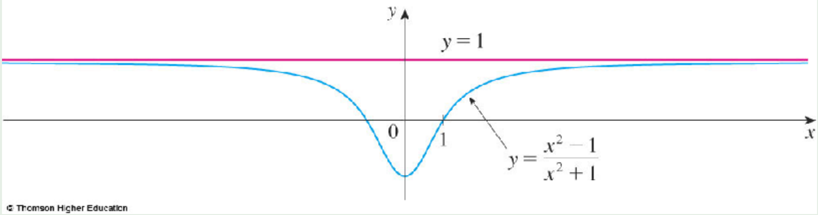
$$\lim_{x\to\infty} f(x) = L \text{ or } \lim_{x\to-\infty} f(x) = L.$$

Example

The function

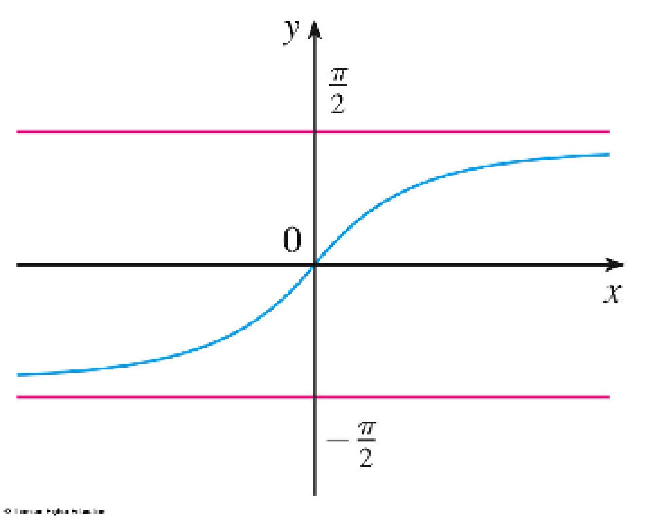
$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

has $y = 1$ as a horiontal asymptote.



Example

The function $f(x) = \tan x$ has two horizontal asymptotes



Infinite limits at infinity

Definition

We write

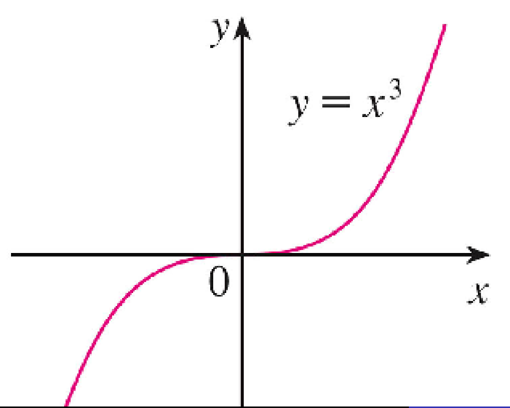
$$\lim_{x \to \infty} f(x) = \infty$$

if the values of $f(x)$ become large as x becomes large.

Example

Let $f(x) = x^3$. Then

$$\lim_{x\to\infty} x^3 = \infty \text{ and } \lim_{x\to\infty} x^3 = \infty.$$



Examples

Compute the following limits

1. $\lim_{x\to\infty} \frac{x^4-x^2+2}{x^3+3}$
2. $\lim_{x\to\infty} \frac{\sqrt{x^4+1}}{2x}$

Dominant Term Rule

Fact

Dominant Term Rule For the limit $\lim_{x\to\infty} P(x)/Q(x)$, *where* $P(x)$ *is a polynomial of degree* n *and* $Q(x)$ *is a polynomial of degree* m,

1. *If* $n < m$, *the limit is* 0,
2. *If* $n > m$, *the limit is* $\pm\infty$,
3. *If* $n = m$, *the limit is the quotient of the coefficients of the highest powers.*