2.7 Derivatives and Rates of Change

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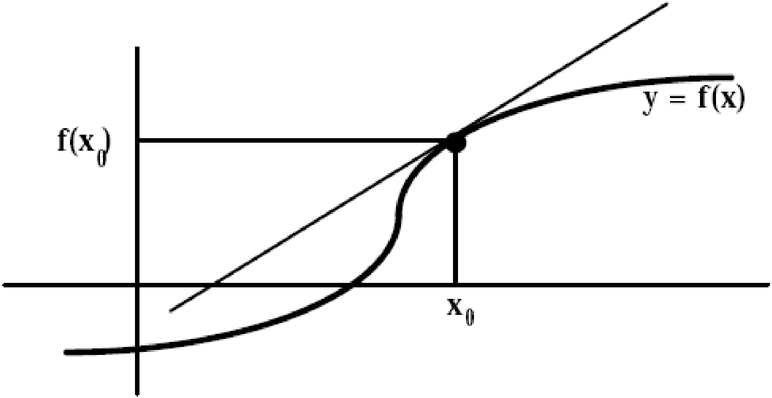
- **The Tangent Line Problem** Given a function $y = f(x)$ defined in an open interval and a point x_0 in the interval, define the tangent line at the point $(x_0, f(x_0))$ on the graph of f.



Example

Find the equations of the tangent lines to the graph of $f(x) = \sqrt{1 - x^2}$ at the points $(0, 1)$ and $(\frac{\sqrt{2}}{2}.\frac{\sqrt{2}}{2})$.

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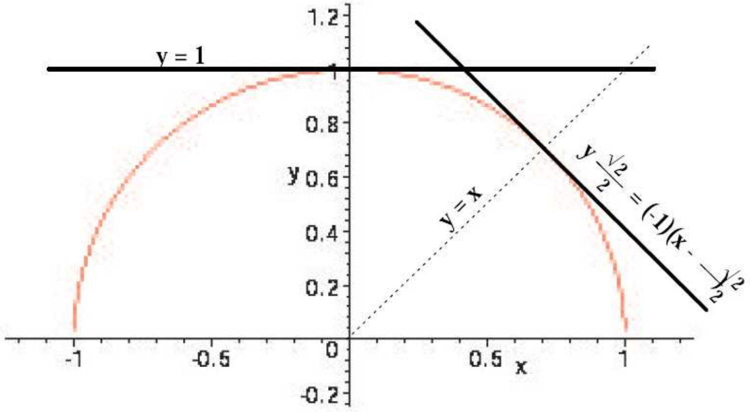
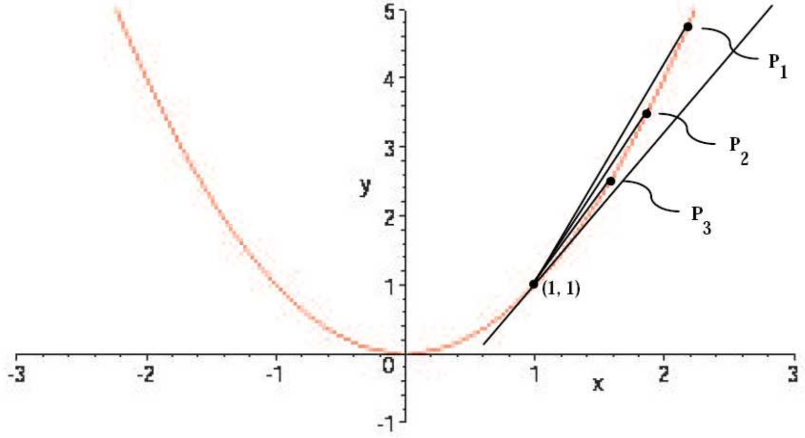


Example

Let $f(x) = x^2$.



The slope of the tangent

Definition

Given a function f and a point x_0 in its domain, the slope of the tangent line at the point $(x_0, f(x_0))$ on the graph of f is

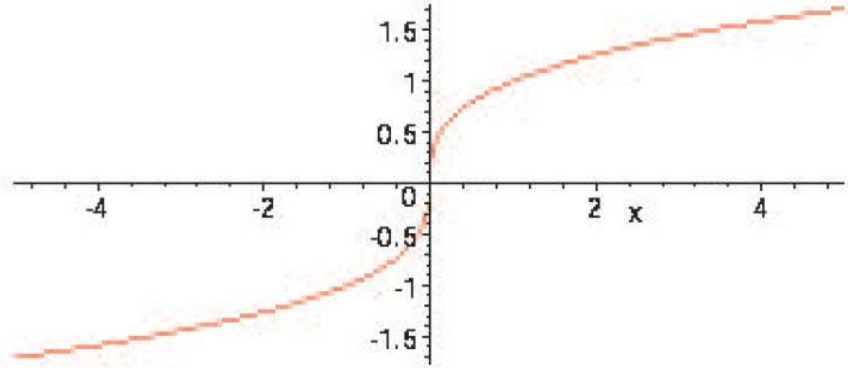
$$m = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$

Example

Given $f(x) = \sqrt{x}$, find the equation of the tangent line at $x = 4$.

Example

Find the tangent line to the graph of $f(x) = x^{1/3}$ at $x = 0$.



Example

Let f be the piecewise defined function

$$f(x) = \begin{cases} 2 - x^2 & x \leq 1 \\ x^3 & x > 1 \end{cases}$$

Is the function continuous, and does it have a tangent line at $x = 1$?

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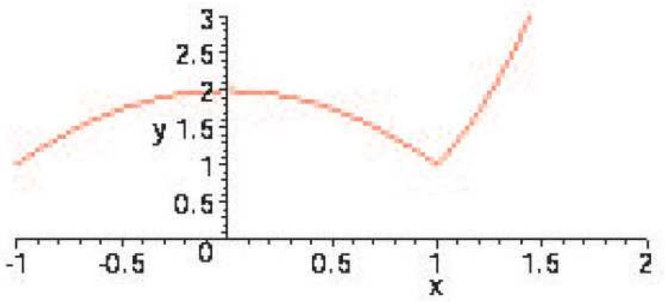

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Equivalently

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

The tangent line (revisited)

Fact

Tangent line The tangent line to $y = f(x)$ *at* $(a, f(a))$ *is the line through* $(a, f(a))$ *whose slope is equal to* $f'(a)$*, the derivative of* f *at* a*.*

Definition

- The average rate of change of y with respect to x over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

Rates of change

Definition

- The average rate of change of y with respect to x over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

- The instantaneous rate of change of y with respect to x is

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

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2. In practical terms, what does it mean to say that $f'(1,000) = 9$?

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1. What is the meaning of the derivative $f'(x)$? What are its units?
2. In practical terms, what does it mean to say that $f'(1,000) = 9$?
3. Which do you think is greater, $f'(50)$ or $f'(500)$? What about $f'(5,000)$?