Coordinates in $\mathbb{R}^2$ and $\mathbb{R}^3$

Lecture 1

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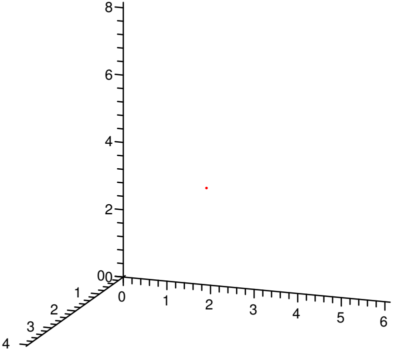
12.1: Three-dimensional coordinate systems



Three-dimensional coordinate systems

- We chose a fixed point O (the origin)
- We also chose three directed lines through O that are perpendicular to each other: **the coordinate axes**
- We label them the x-axis, y-axis, and z-axis.

Three-dimensional coordinate systems

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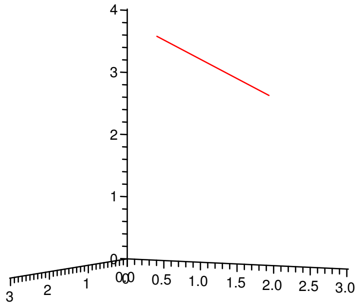
- A point P in space is represented by a triple (a, b, c)
- a is the x-coordinate
- b is the y-coordinate
- c is the z-coordinate
- This correspondence between points and triples (a, b, c) in $\mathbb{R}^3$ is called a three dimensional rectangular coordinate system.

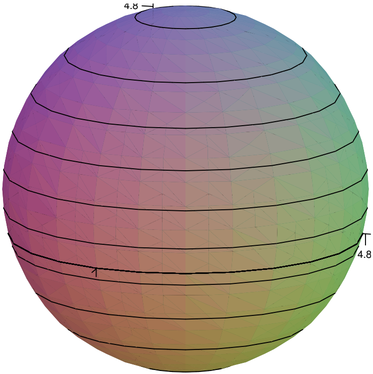
Distance between two points



Distance formula

- The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P(x_2, y_2, z_2)$ is
$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equation of a sphere



Equation of a Sphere

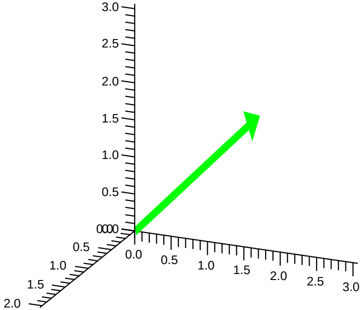
- An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2.$$

- If the center is the origin

$$x^2 + y^2 + z^2 = r^2.$$

- A **vector** has **initial point** A and **terminal point** B
- We write $\vec{AB}$ or $\vec{u}$ or $\mathbf{u}$.



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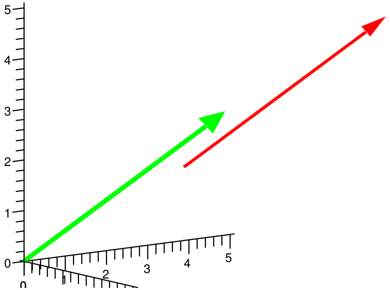

Vector Addition

- If $\mathbf{u}$ and $\mathbf{v}$ are vectors positioned so the initial point of $\mathbf{v}$ is at the terminal point of $\mathbf{u}$, then the **sum** $\mathbf{u} + \mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.

Scalar multiplication

- If c is a scalar and $\mathbf{v}$ is a vector, then the **scalar multiple** $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of $\mathbf{v}$ and whose direction is the same as $\mathbf{v}$ if $c > 0$ and is opposite to $\mathbf{v}$ if $c < 0$.

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- We call $-\mathbf{v}$ the **negative** of $\mathbf{v}$.
- The **difference** $\mathbf{u} - \mathbf{v}$ of two vectors is

$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$$

Components

- If we place the initial point of a vector **a** at the origin, then the terminal point of **a** has coordinates of the form (a_1, a_2) or (a_1, a_2, a_3).

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- These coordinates are called **components** of **a**

$$\mathbf{a} = \langle a_1, a_2 \rangle \quad \text{or} \quad \mathbf{a} = \langle a_1, a_2, a_3 \rangle.$$

The length of a vector

Definition

The **length** of a vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

Standard basis vectors

Definition

The vectors

$$\mathbf{i} = \langle 1, 0, 0 \rangle, \ \mathbf{j} = \langle 0, 1, 0 \rangle, \ \mathbf{k} = \langle 0, 0, 1 \rangle$$

are called the **standard basis vectors**.