The Chain Rule

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Notes

The Chain Rule (case 1)

Definition

- Suppose that $z = f(x, y)$ is a differentiable function of x and y, where $x = g(t)$ and $y = h(t)$ are both differentiable functions of t. Then z is a differentiable function of t and

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial f}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial f}{\partial y}\frac{\mathrm{d}x}{\mathrm{d}t}.$$

Notes

Examples

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- If $z = x^2y + xy^3$, where $x = \cos t$, $y = \sin t$, find $\mathrm{d}z/\mathrm{d}x$ when $t = \pi/2$.
- Find $\mathrm{d}z/\mathrm{d}t$ if $z = \sqrt{x^2 + y^2}$ and $x = e^{2t}$ and $y = e^{-2t}$.
- The pressure P (in kilopascals), volume V (in liters), and temperature T (in kelvins) of a mole of an ideal gas are related by the equation $PV = 8.31T$. Find the rate at which the pressure is changing when the temperature is $300K$ and increasing at a rate of $0.1K/s$ and the volume is $100\ L$ and increasing at a rate of $0.2\ L/s$.

Notes

The Chain Rule (Case 2)

Definition

- Suppose t hat $z = f(x, y)$ is a differentiable function of x and y, where $x = g(s, t)$ and $y = h(s, t)$ are differentiable functions of s and t. Then

$$\begin{aligned}
\frac{\partial z}{\partial s} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \\
\frac{\partial z}{\partial t} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}
\end{aligned}$$

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Examples

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Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ for the following examples:

- $z = e^{xy} \sin x$, where $x = 2s + 4t$, $y = \frac{2s}{3t}$.
- $z = \ln(x^2 + y^2)$, where $x = e^s \cos t$ and $y = e^s \sin t$.
- $w = xy + xz + yz$, where $x = st$, $y = e^{st}$, $z = x + t$.

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