

Notes

Notes

Theorem

Suppose that f is a differentiable function of two (or three) variables. The maximum value of the directional derivative $D_{\mathbf{u}}f(x, y)$ is $|\nabla f|$ and it occurs when \mathbf{u} has the same direction as the gradient vector $\nabla f(x)$.

Marius Ionescu ()	Directional Derivatives and the Gradient	October 26, 2012	3 / 12
Example			
Example			
Example			
Example If $f(x, y) = xe^{y}$	find the rate of change of f at t	he point $P(2,0)$ is	n
• If $f(x, y) = xe^{y}$, find the rate of change of f at t om P to $Q(rac{1}{2},2).$	he point <i>P</i> (2,0) i	n
• If $f(x, y) = xe^{y}$ the direction from	-		

Example

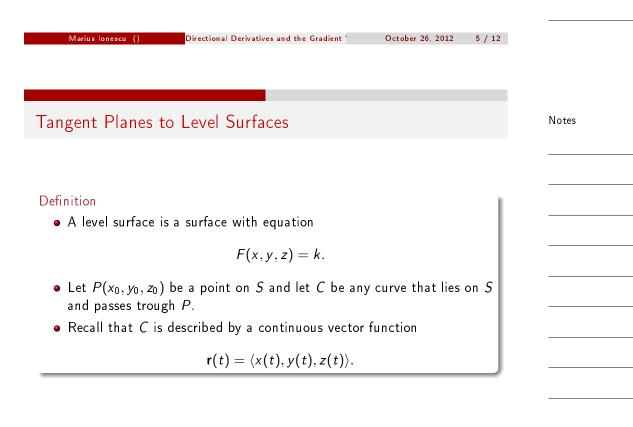
Example

Suppose that the temperature at a point (x, y, z) in space is given by

$$T(x, y, z) = \frac{80}{1 + x^2 + 2y^2 + 3z^2}$$

where T is measured in degree Celsius and x, y, z in meters.

- In which direction does the temperature increase fastest at the point (1, 1, -2)?
- What is the maximum rate of increase?



Tangent Planes to Level Surfaces

Notes

Fact

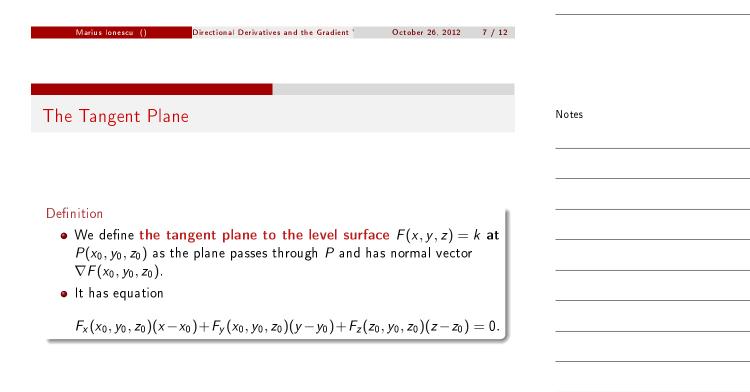
• If x, y, and z are differentiable and F is also differentiable, we can apply the Chain Rule:

$$\frac{\partial F}{\partial x}\frac{dx}{dt} + \frac{\partial F}{\partial y}\frac{dy}{dt} + \frac{\partial F}{\partial z}\frac{dz}{dt} = 0;$$

• Or

$$\nabla F \cdot \mathbf{r}'(t) = 0$$

 The gradient vector at P, ∇F(x₀, y₀z₀) is perpendicular to the tangent vector r'(t₀) to any curve C on S that passes through P.



The Normal Line

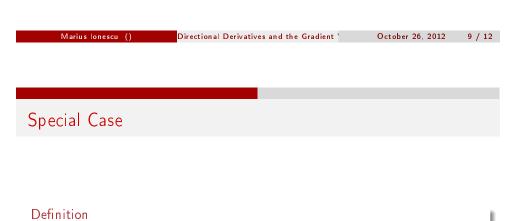
Notes

Notes

Definition

- The normal line to S at P is the line passing through P and perpendicular to the tangent plane.
- The symmetric equations are

$$\frac{x - x_0}{F_x(x_0, y_0, z_0)} = \frac{y - y_0}{F_y(x_0, y_0, z_0)} = \frac{z - z_0}{F_z(x_0, y_0, z_0)}$$



If the equation of the surface S is of the form z = f(x, y), that is

$$F(x, y, z) = f(x, y) - z = 0$$

then the equation of the tangent plane becomes

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - z_0) = 0.$$

Examples

Notes

Examples

• Find the tangent plane and normal line of the surface

$$F(x, y, z) = x^2 + y^2 + z - 9 = 0$$

at the point $P_0(1, 2, 4)$.

 $\bullet\,$ Find the equation of the tangent plane at the point (-2,1,-3) to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3.$$



Significance of the Gradient Vector

Fact

- The gradient ∇f gives the direction of fastest increase of f.
- The gradient ∇f is orthogonal to the level surface S of f through a point P.