15.2: Iterated Integrals

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Iterated Integrals

Definition

Let f be a function of two variables that is integrable on the rectangle $R = [a, b] \times [c, d]$.

There are two iterated integrals

$$\int_a^b \int_c^d f(x, y) dy dx$$

and

$$\int_c^d \int_a^b f(x, y) dx dy.$$

Example

Example

Evaluate the iterated integrals

Example

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- $\int_0^2 \int_1^2 x^2 y \, dy \, dx$

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- $\int_0^2 \int_1^2 x^2 y\, dy\, dx$
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Fubini's Theorem

Theorem

If f is continuous on the rectangle $R = [a, b] \times [c, d]$, then

$$\iint_R f(x, y)dA = \int_a^b \int_c^d f(x, y)dydx = \int_c^d \int_a^b f(x, y)dxdy.$$

Examples

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Calculate the following integrals

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- $\iint_R (1 - xy^2) dA$, where $R = [0, 4] \times [-1, 2]$

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- $\iint_R (1 - xy^2) dA$, where $R = [0,4] \times [-1,2]$
- $\iint_R (x - 3y^2) dA$, where $R = [0,2] \times [1,2]$.

Examples

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Calculate the following integrals

- $\iint_R (1 - xy^2) dA$, where $R = [0, 4] \times [-1, 2]$
- $\iint_R (x - 3y^2) dA$, where $R = [0, 2] \times [1, 2]$.
- $\iint_R \frac{1+x^2}{1+y^2} dA$, where $R = [0, 1] \times [0, 1]$.

Examples

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Calculate the following integrals

- $\iint_R (1 - xy^2) dA$, where $R = [0, 4] \times [-1, 2]$
- $\iint_R (x - 3y^2) dA$, where $R = [0, 2] \times [1, 2]$.
- $\iint_R \frac{1+x^2}{1+y^2} dA$, where $R = [0, 1] \times [0, 1]$.
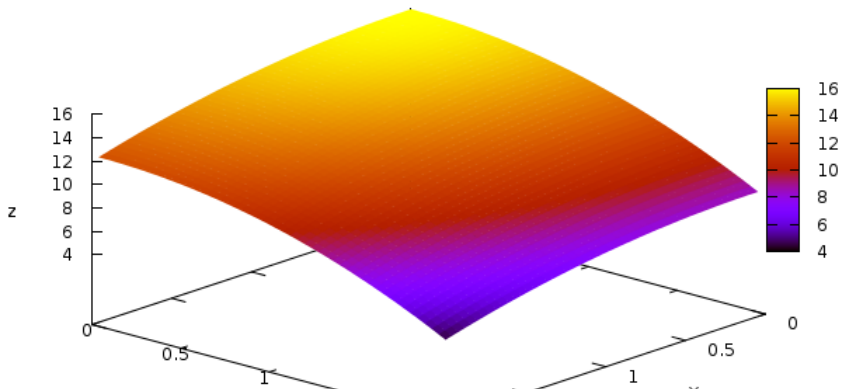- $\iint_R xe^{-xy} dA$, where $R = [0, 3] \times [0, 2]$.

Examples

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Calculate the following integrals

- $\iint_R (1 - xy^2)dA$, where $R = [0, 4] \times [-1, 2]$
- $\iint_R (x - 3y^2)dA$, where $R = [0, 2] \times [1, 2]$.
- $\iint_R \frac{1+x^2}{1+y^2} dA$, where $R = [0, 1] \times [0, 1]$.
- $\iint_R xe^{-xy} dA$, where $R = [0, 3] \times [0, 2]$.
- $\iint_R \cos(x - y)dA$, where $R = [0, \pi/2] \times [0, \pi/2]$.

Example

Example

Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$, and the coordinate planes.

Example

Example

Find the volume of the solid that lies under the hyperbolic paraboloid $z = 3y^2 - x^2 + 2$ and above the rectangle $R = [-1, 1] \times [1, 2]$.

3*y^2-x^2+2