

15.2: Iterated Integrals

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Notes

Iterated Integrals

Definition

Let f be a function of two variables that is integrable on the rectangle $R = [a, b] \times [c, d]$.

There are two iterated integrals

$$\int_a^b \int_c^d f(x, y) dy dx$$

and

$$\int_c^d \int_a^b f(x, y) dx dy.$$

Notes

Example

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Evaluate the iterated integrals

- $\int_0^2 \int_1^2 x^2 y dy dx$
- $\int_1^2 \int_0^2 x^2 y dx dy$

Notes

Fubini's Theorem

Theorem

If f is continuous on the rectangle $R = [a, b] \times [c, d]$, then

$$\iint_R f(x, y) dA = \int_a^b \int_c^d f(x, y) dy dx = \int_c^d \int_a^b f(x, y) dx dy.$$

Notes

Examples

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Calculate the following integrals

- $\iint_R (1 - xy^2) dA$, where $R = [0, 4] \times [-1, 2]$
- $\iint_R (x - 3y^2) dA$, where $R = [0, 2] \times [1, 2]$.
- $\iint_R \frac{1+x^2}{1+y^2} dA$, where $R = [0, 1] \times [0, 1]$.
- $\iint_R xe^{-xy} dA$, where $R = [0, 3] \times [0, 2]$.
- $\iint_R \cos(x - y) dA$, where $R = [0, \pi/2] \times [0, \pi/2]$.

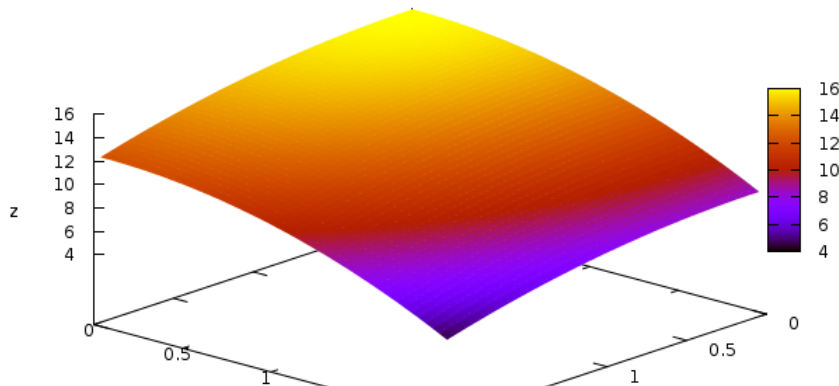
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Example

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Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$, and the coordinate planes.

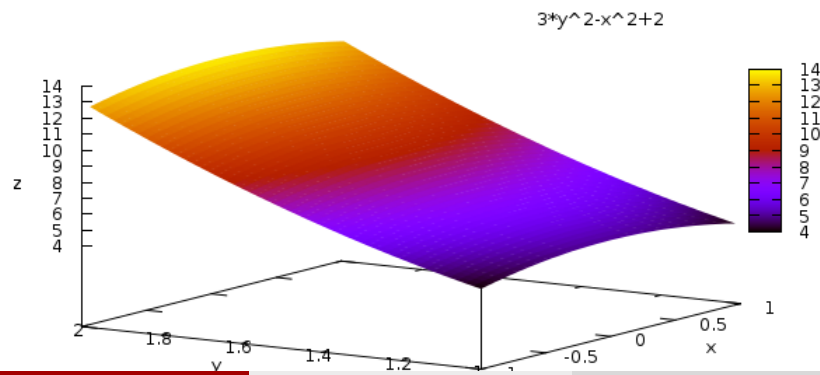
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Example

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Find the volume of the solid that lies under the hyperbolic paraboloid $z = 3y^2 - x^2 + 2$ and above the rectangle $R = [-1, 1] \times [1, 2]$.



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