The Cross Product

Lecture 3

September 7, 2012

Properties

Given two vectors **a** and **b** we want to build a vector $\mathbf{a} \times \mathbf{b}$ called the **cross product** of **a** and **b** with the following properties:

- The vector $\mathbf{a} \times \mathbf{b}$ is orthogonal to both **a** and **b**.
- If θ is the angle between **a** and **b** then

$$|a \times b| = |a||b| \sin \theta$$

- Two nonzero vectors are parallel if and only if $\mathbf{a} \times \mathbf{b} = 0$
- The length of the cross product $\mathbf{a} \times \mathbf{b}$ is equal to the area of the parallelogram determined by **a** and **b**.

The Cross Product

- If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **cross product** of $\mathbf{a}$ and $\mathbf{b}$ is the vector

$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

Examples

- Find the crossed product of the vectors $\langle -1, 2, 1\rangle$ and $\langle 1, -2, 2\rangle$.
- Find the crossed product of the vectors $\langle \sqrt{2}, -\sqrt{2}, 1\rangle$ and $\langle 1/2, 1, 1\rangle$.

Examples

- Find a vector perpendicular to both $\langle -2, 2, 0\rangle$ and $\langle 0, 1, 2\rangle$ of the form $\langle 1, \underline{\hspace{2em}}, \underline{\hspace{2em}}\rangle$
- Find the area of the triangle with vertices $P(0, 0, 0)$, $Q(-2, 2, 5)$, $R(0, 3, -3)$.

Other Properties of the Cross Product

- You can check Theorem 11 on page 836 for the complete list of the properties of the cross product of vectors. I will mention only one:
- $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$.

The volume of a parallelepiped

- The volume of a parallelepiped determined by the vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ is the magnitude of their scalar triple product:

$$V = |a \cdot (b \times c)|$$

- Find the volume of the parallelepiped with adjacent edges PQ, PR, PS where$P(1, 4, -3)$, $Q(3, 7, 0)$, $R(0, 3, -4)$, $S(7, 2, -1)$.