14.1 Functions of Several Variables

October 1, 2012

Functions of two variables

Functions of two variables

- A function of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$.

Functions of two variables

- A function of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$.
- The set D is the domain of f and its range is the set of values that f takes on.

Functions of two variables

- A function of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$.
- The set D is the domain of f and its range is the set of values that f takes on.
- We also write $z = f(x, y)$

Functions of two variables

- A function of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$.
- The set D is the domain of f and its range is the set of values that f takes on.
- We also write $z = f(x, y)$
- The variables x and y are **independent variables** and z is the **dependent variable**.

Examples

Examples

Examples

Examples

- Find the domain of the function

$$f(x,y) = \frac{2x+3y}{x^2+y^2-9}$$

Examples

Examples

- Find the domain of the function

$$f(x, y) = \frac{2x + 3y}{x^2 + y^2 - 9}$$

- Find the domain and range of

$$f(x, y) = \sqrt{4 - x^2 - y^2}$$

Graphs

Definition

- If f is a function of two variables with domain D, then the graph of f is the set of all points $(x, y, z) \in \mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D.

Examples

Example

Examples

Example

- A **linear function** is a function

$$f(x) = ax + by + c$$

Examples

Example

- A **linear function** is a function

$$f(x) = ax + by + c$$

- The graph of such a function is a plane.

Examples

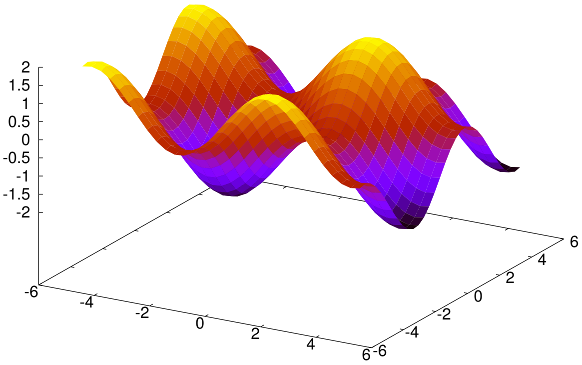
Example

- $f(x, y) = \sin(x) + \sin(y)$

Examples

Example

- $f(x,y) = \sin(x) + \sin(y)$



Examples

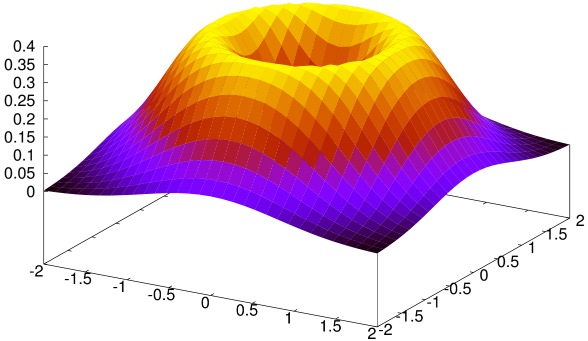
Example

- $f(x, y) = (x^2 + y^2)e^{-x^2-y^2}$

Examples

Example

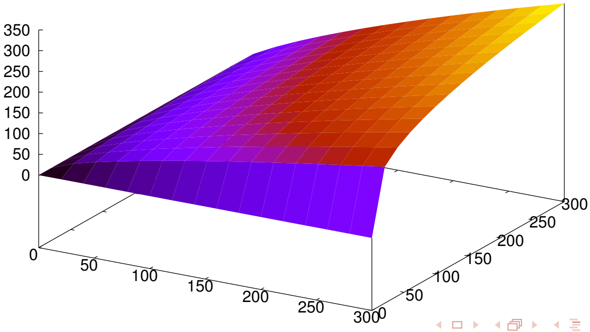
- $f(x, y) = (x^2 + y^2)e^{-x^2-y^2}$



The Cobb-Douglas production function

Example

- $P(L, K) = bL^{\alpha}K^{1-\alpha}$



Level Curves

Definition

- The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is constant.

Examples

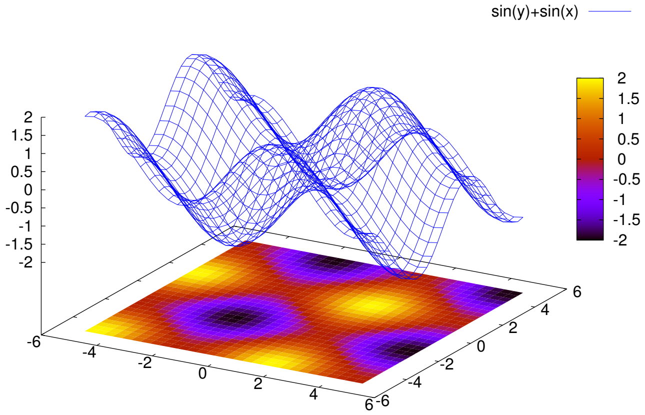
Example

- $f(x, y) = \sin(x) + \sin(y)$

Examples

Example

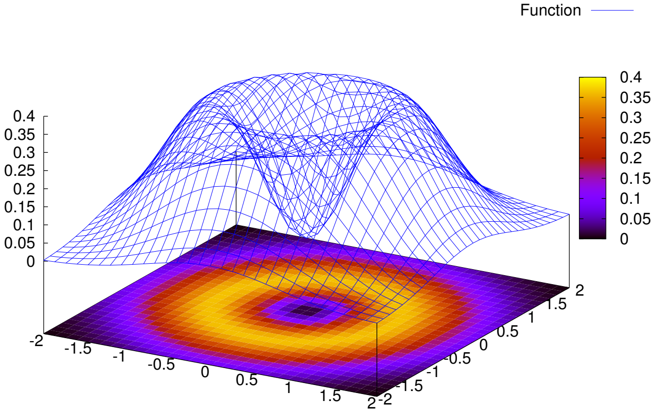
- $f(x, y) = \sin(x) + \sin(y)$



Examples

Example

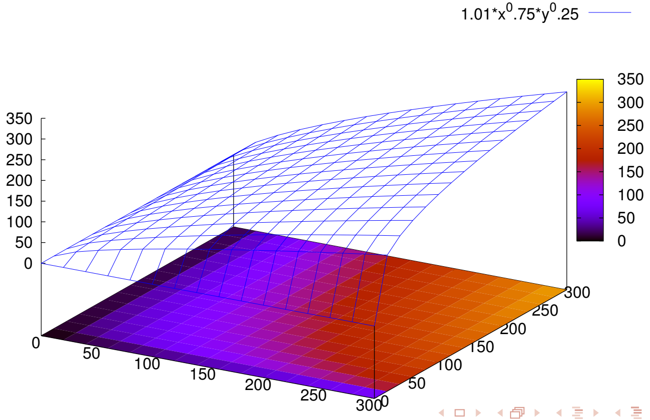
- $f(x,y) = (x^2 + y^2)e^{-x^2-y^2}$



The Cobb-Douglas production function

Example

- $P(L, K) = 1.01L^{0.75}K^{0.25}$



14.2 Limits and Continuity

Definition

The **limit** of $f(x, y)$ as (x, y) approaches (a, b) is L and we write

$$\lim_{(x,y)\to(a,b)} f(x, y) = L$$

if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that if $(x, y) \in D$ and $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$ then $|f(x, y) - L| < \varepsilon$. We also write $f(x, y) \to L$.

Definition

The **limit** of $f(x, y)$ as (x, y) approaches (a, b) is L and we write

$$\lim_{(x,y)\to(a,b)} f(x, y) = L$$

if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that if $(x, y) \in D$ and $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$ then $|f(x, y) - L| < \varepsilon$.
We also write $f(x, y) \to L$.

WHAT?

It is easier to show that a function does not have a limit!

Fact

If $f(x,y) \to L_1$ *as* $(x,y) \to (a,b)$ *along a path* C_1*and* $f(x,y) \to L_2$ *as* $(x,y) \to (a,b)$ *along a path* C_2*, where* $L_1 \neq L_2$*, then* $\lim_{(x,y)\to(a,b)} f(x,y)$ *does not exist.*

Examples

Examples

Show that the following limits do not exist:

Examples

Examples

Show that the following limits do not exist:

- $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$.

Examples

Examples

Show that the following limits do not exist:

- $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$.
- $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$.

Continuity

Definition

Continuity

Definition

- A function f of two variables is called **continuous at** (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$$

Continuity

Definition

- A function f of two variables is called **continuous at** (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$$

- **Examples:** polynomials, rational, trigonometric, exponential, logarithmic functions are continuous on theirs domain.

Examples

Example

Examples

Example

- Find the largest set on which the function

$$\frac{2xy}{9 - x^2 - y^2}$$

is continuous.