

Examples

Examples

- Find the domain of the function

$$f(x, y) = \frac{2x + 3y}{x^2 + y^2 - 9}$$

- Find the domain and range of

$$f(x, y) = \sqrt{4 - x^2 - y^2}$$

Notes

()

Graphs

Definition

- If f is a function of two variables with domain D , then the graph of f is the set of all points $(x, y, z) \in \mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D .

Notes

()

Examples

Example

- A **linear function** is a function

$$f(x) = ax + by + c$$

- The graph of such a function is a plane.

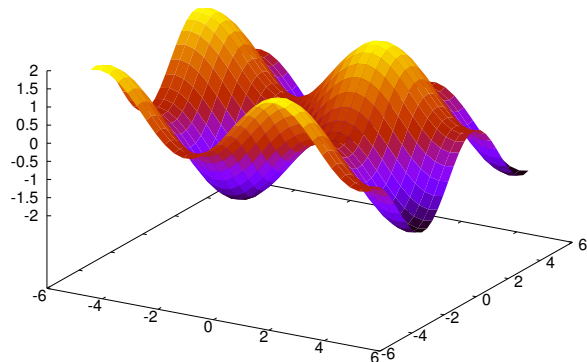
Notes

()

Examples

Example

- $f(x, y) = \sin(x) + \sin(y)$



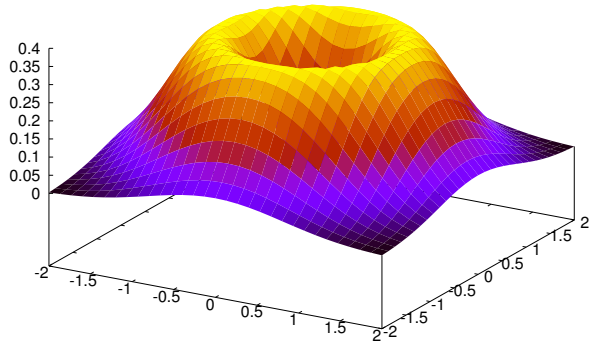
Notes

()

Examples

Example

- $f(x, y) = (x^2 + y^2)e^{-x^2 - y^2}$



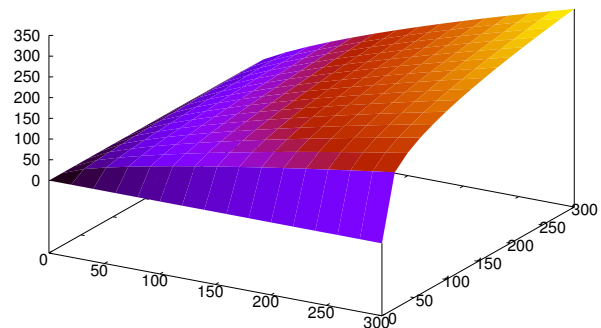
()

Notes

The Cobb-Douglas production function

Example

- $P(L, K) = bL^\alpha K^{1-\alpha}$



()

Notes

Level Curves

Notes

Definition

- The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is constant.

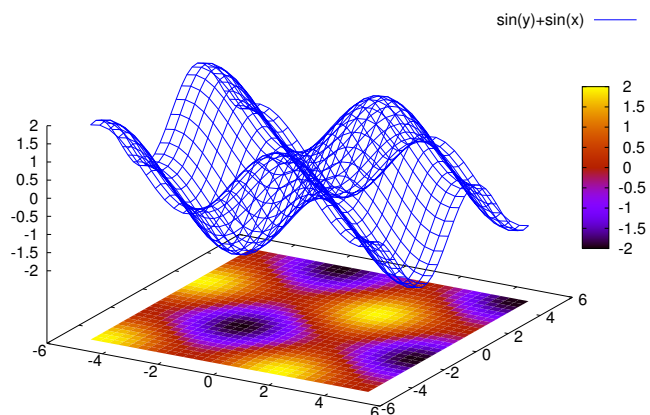
()

Examples

Notes

Example

- $f(x, y) = \sin(x) + \sin(y)$

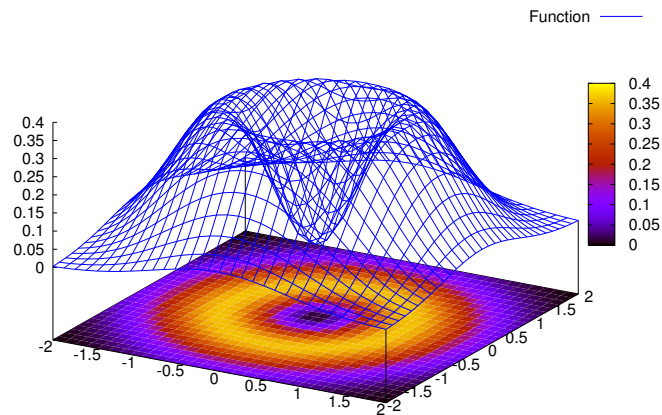


()

Examples

Example

- $f(x, y) = (x^2 + y^2)e^{-x^2 - y^2}$



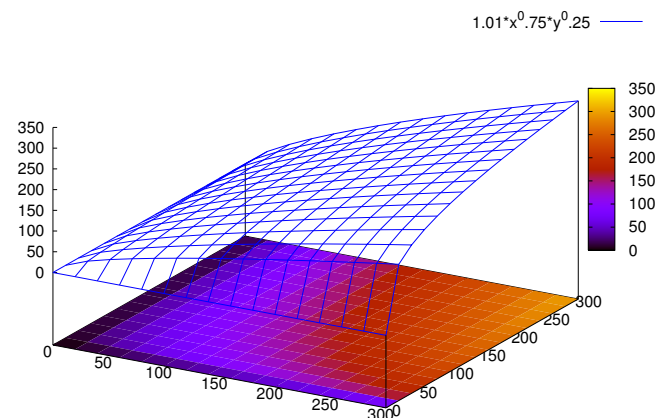
()

Notes

The Cobb-Douglas production function

Example

- $P(L, K) = 1.01L^{0.75}K^{0.25}$



()

Notes

14.2 Limits and Continuity

Definition

The **limit** of $f(x, y)$ as (x, y) approaches (a, b) is L and we write

$$\lim_{(x,y) \rightarrow (a,b)} f(x, y) = L$$

if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that if $(x, y) \in D$ and $0 < \sqrt{(x - a)^2 + (y - b)^2} < \delta$ then $|f(x, y) - L| < \varepsilon$. We also write $f(x, y) \rightarrow L$.

WHAT?

Notes

()

14.1 Functions of Several Variables

October 1, 2012

13 / 17

It is easier to show that a function does not have a limit!

Fact

If $f(x, y) \rightarrow L_1$ as $(x, y) \rightarrow (a, b)$ along a path C_1 and $f(x, y) \rightarrow L_2$ as $(x, y) \rightarrow (a, b)$ along a path C_2 , where $L_1 \neq L_2$, then $\lim_{(x,y) \rightarrow (a,b)} f(x, y)$ does not exist.

Notes

()

14.1 Functions of Several Variables

October 1, 2012

14 / 17

Examples

Examples

Show that the following limits do not exist:

- $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$.
- $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$.

Notes

()

14.1 Functions of Several Variables

October 1, 2012

15 / 17

Continuity

Definition

- A function f of two variables is called **continuous at** (a, b) if

$$\lim_{(x,y) \rightarrow (a,b)} f(x, y) = f(a, b)$$

- **Examples:** polynomials, rational, trigonometric, exponential, logarithmic functions are continuous on their domain.

Notes

()

14.1 Functions of Several Variables

October 1, 2012

16 / 17

