

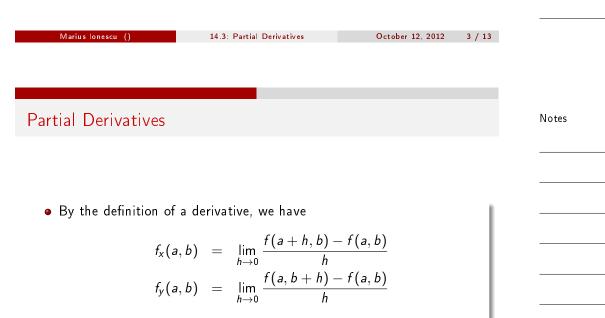
Partial derivative of f with respect to y

Notes

Definition

- Now keep x = a fix.
- Let h(y) = f(a, y).
- If h has a derivative at b, then we call it the partial derivative of f with respect to y at (a, b)

$$f_y(a,b) = h'(b)$$



• The partial derivatives of f(x, y) are the functions $f_x(x, y)$ and $f_y(x, y)$ obtained by letting the point (a, b) vary.

Notes

• If
$$z = f(x, y)$$
, we write
 $f_x(x, y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}f(x, y) = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f$
 $f_y(x, y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}f(x, y) = \frac{\partial z}{\partial y} = f_2 = D_2 f = D_y f$

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Rule for Finding Partial Derivatives of z = f(x, y)

Notes

- To find f_x regard y as a constant and differentiate f(x, y) with respect to *x*.
- To find f_y regard x as a constant and differentiate f(x, y) with respect to *y*.

October 12, 2012 5 / 13

Examples

Notes

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Examples

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• If
$$f(x, y) = x^2 + 3x^3y - xy^2$$
 find $f_x(0, 1)$ and $f_y(1, 0)$
• Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the functions
• $f(x, y) = \frac{2y}{y + \cos x}$
• $f(x, y) = e^{x^2 + y^2 + 1}$

$$f(x,y) = \ln(x+y)$$

Examples (cont'd) Example • Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by	Marius Ionescu ()	14.3: Partial Derivatives	October 12, 2012	7 / 13
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 $x^3 + y^3 + z^3 + 6xyz = 1.$

Notes

- Partial derivative can be interpreted as rates of change.
- The geometric interpretation: the partial derivatives are the slopes of the tangent lines at P(a, b, c) to the curves given by the intersection of the surface given by z = f(x, y) and the planes x = a and y = b.

Higher Derivatives

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Definition

• If f is a function of two variables, then its partial derivatives f_x and f_y are also functions of two variables.

14.3: Partial Derivatives

- So why stop here?
- The second partial derivatives of f are

$$(f_{x})_{x} = f_{xx} = f_{11} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial^{2} f}{\partial^{2} x} = \frac{\partial^{2} z}{\partial^{2} x}$$
$$f_{xy} = \frac{\partial^{2} f}{\partial x \partial y} = \cdots$$
$$f_{yx} = \frac{\partial^{2} f}{\partial y \partial x} = \cdots$$
$$f_{yy} = \frac{\partial^{2} f}{\partial^{2} y}$$

Notes

October 12, 2012

9 / 13

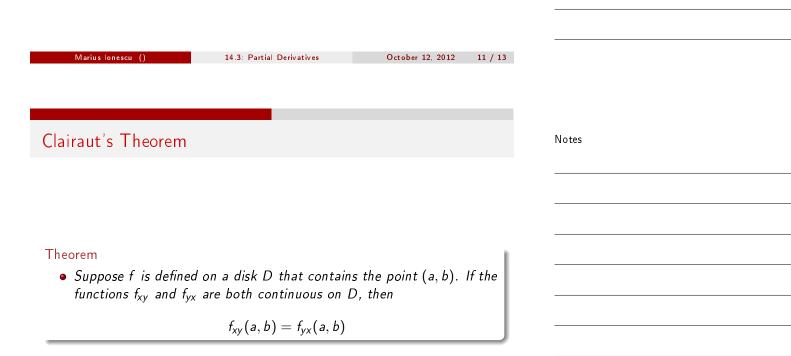
Example

Notes

Example

• Find the second derivatives of

 $f(x, y) = x^3 + x^2 y^3 - 2y^2$



Partial derivatives of order 3 and higher

Notes

Examples

- Calculate f_{xxy} if $f(x, y) = \sin(3x^2 + xy)$.
- Find the partial derivatives of

$$f(x,y) = \int_x^y e^{t^2 + t + 1} \mathrm{d}t$$

• Find $f_x, f_{y,}, f_{xy}, f_{yx}$ for

$$f(x,y) = xye^{3xy}$$

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Notes